This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

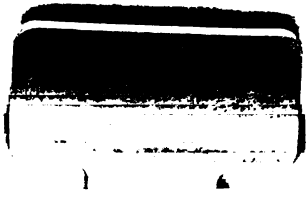
Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

UC-NRLF
$B 32 456

Digitized by Google

LIBRARY
OF THE
UNIVERSITY OF CALIFORNIA.

Class



Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

Aud

GENERAL

VOL. II. No. 3.

SELECTED PAPERS

—OF THE—

Rensselaer Society of Engineers.

TROY, N. Y.

EDITED BY THE COMMITTEE ON PUBLICATION.

JANUARY, 1894.

CONTENTS:

THEORY OF THE MASONRY ARCH.....WM. H. BURR

NOTES ON RAILWAY EARTHWORK.....W. G. RAYMOND

DETERMINATION OF THE PROPER SIZE OF THE SUPPLY MAIN FOR THE NEW WATER SUPPLY OF GALVESTON, TEXAS.....WYNKOOP KIERSTED

NATURAL ASPHALTUM AND ITS COMPOUNDS....J. W. HOWARD

TROY, N. Y.:
E. H. LISK, PRINTER, 502 FULTON ST.
1894.

11774

Digitized by Google

Digitized by Google

VOL. II. No. 3.

SELECTED PAPERS

—OF THE—

Rensselaer Society of Engineers.

TROY, N. Y.

EDITED BY THE COMMITTEE ON PUBLICATION.

JANUARY, 1894.

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

CONTENTS:

THEORY OF THE MASONRY ARCH.....WM. H. BURR

NOTES ON RAILWAY EARTHWORK.....W. G. RAYMOND

DETERMINATION OF THE PROPER SIZE OF THE SUPPLY MAIN FOR THE NEW WATER SUPPLY OF GALVESTON, TEXAS.....WYNKOOP KIERSTED

NATURAL ASPHALTUM AND ITS COMPOUNDS....J. W. HOWARD

TROY, N. Y.:
E. H. LISK, PRINTER, 502 FULTON ST.
1894.

Digitized by Google

tinue to stand with no evident sign of insecurity, though such a condition of things is directly in contradiction to the results given by the treatment.

A proper consideration of all the influences existing will not demonstrate the instability of a structure which stands with every evidence of security.

In the treatment which follows, the ring stones will be considered to be unchangeable in shape, or perfectly rigid. This hypothesis eliminates from the question all consideration of the condition of the ends of the arch at the skew-backs, and is not only admissible, but necessary, as the nearest approach to the actual condition of things. There is probably no arch in existence in which perfect continuity from skew-back to skew-back has been preserved after the removal of the centre. If the arch is assumed to be elastic, and fixed at the ends, this continuity must not only be absolutely perfect and extend through the skew-backs, but the whole structure must settle uniformly. If the ends are assumed to be hinged, continuity must hold between the skew-backs, and uniform settlement must take place. In consequence of the number of joints which are observed to open in many existing arches, even upon hasty examination, it is in the highest degree probable that neither perfect continuity nor perfectly uniform settlement occurs in any case.

It is, therefore, necessary to make the case one of pure statics only, instead of combined statics and strains.

It is supposed that no special devices, such as metallic clamps, are used to bind together the ring stones. With such appliances continuity and attendant conditions, might at least approximately be attained.

It is also to be assumed that no such devices are used to secure the ring stones to the spandril walls or filling. For the reasons already given, then, all forces acting on the arch ring must be directed inward, or in other words they must be pressures. The cement joint or joints between the ring stones and spandril can not be considered capable of resisting tension. It is not implied that good hydraulic cement mortar cannot resist tension. But it *is* true that if a mortar joint in an arch is capable of resisting any tension at all, it is impossible to determine to what extent it can be relied upon in that capacity; and in some joints, in good work, even contact does not exist. The real condition of things for a masonry arch will be more nearly

Digitized by Google

described, therefore, by assuming that the arch ring is subject to *pressures* only. It then becomes necessary to determine just what these pressures are.

The vertical loading of an arch consists of two parts: the fixed, or permanent load and the moving load. The fixed load includes the spandril filling, or backing, on which the moving load and a part of the permanent load is applied, and which rests directly on the ring stones. The arch ring, therefore, must receive all pressure from and through the spandril.

The resultant pressure on the arch ring may be decomposed into two parts, one of which is the vertical loading, and the other is called the "conjugate" pressure. In many cases, perhaps the majority, this conjugate pressure is horizontal, but in others it is not. In Fig. 6, if the backing $B\ D\ C$ and $C\ E\ F$ is composed of dry earth, of which $B\ E$ is the trace of the plane upper surface, it is known from the theory of internal stress, or earth pressure, that the conjugate pressures h_1, h_2, h_3, etc., against the different ring stones will be parallel to that upper surface $B\ E$. The resultant pressure against any stone, as K, will be that force whose components are h_2, and the vertical load comprised between the two vertical lines v and v' drawn from the joints between K and the two adjacent stones, combined with the weight of the stone itself. In other words the vertical load is simply the weight resting upon the stone in question, and this is to be added to that of the stone itself. This weight is a fixed quantity depending upon the loading; but the conjugate pressure (as h_2) *may have any value between the conjugate pressure of earth loaded with its own weight only, and the abutting power of earth.* This is a principle of great importance and it will be seen to have a very potent influence on the determination of the line of resistance.

Any conjugate pressure (as h_2) on the posterior surface of a ring stone is simply *the conjugate earth pressure exerted on that portion of a vertical plane included between two conjugate planes drawn through the upper extremities* (*as a and b*) *of the two bounding joints.*

These methods of determining the components of the resultant force acting on any stone are very simple and very important. In the figure $a\ c$ is vertical and $b\ c$ parallel to $B\ E$, but the methods just given are in no manner dependent upon those directions.

Digitized by Google

If the backing $B\ D\ C$ and $C\ E\ F$, (Fig. 6, Plate I.) is of masonry or any other tenacious material, the direction of the conjugate pressure cannot be determined; it will depend upon the nature of the material, character of work, etc. It *may* be parallel to the conjugate direction $B\ E$, but it will probably not be.

These conjugate pressures may, therefore, in this case be assumed to have any directions whatever which will give a line of resistance most favorable to stability, since this line of resistance may be supposed to exist at the instant before rupture. It is implied, of course, that the obliquities of these pressures, in reference to the surfaces on which they act, must not exceed the limiting angles of friction, or the angles of repose, for those surfaces.

In regard to the magnitudes of these pressures, they may have any values which will cause their greatest intensities to lie between the limits of zero and the working resistance to compression of either the backing or ring stones. The upper limit of the intensity will be the least of the two working resistances. If it were a mere question of statics, aside from the consideration of a safety factor, "ultimate resistance" would be used instead of "working resistance."

Thus, *within certain limits*, there is a capacity of adjustment, requisite to the conditions of stability, of the pressures h between the ring stones and backing.

It should be observed at this point that, if the backing is of good masonry and well bonded to the ring stones, the line of resistance may be located anywhere, within safe limits, between the upper surface of the backing and the soffit of the arch. In such a case, questions in regard to the stability can hardly arise; it (stability) is secured almost *a priori*.

There have, then, thus far, been shown the methods for finding the vertical and other component of the resultant pressure on each stone of the arch ring; and it is to be carefully remembered that this resultant includes the weight of the stone itself. These resultant pressures are simply the external forces acting upon the individual stones; and they are completely known, for when the character of the pressures h_1, h_2, etc., is once determined, their centres are easily found by known means. The lines of action of the vertical loading and weight, for each stone, are always known.

Digitized by Google

Now, let *any* portion of the arch, as $C\ L\ M\ G$, be considered. If *all* the external forces acting on this portion were *completely* known, a trial line of resistance could at once be constructed. But even if the external forces acting on the stones are given, those which act in the joints $C\ G$ and $L\ M$ are still unknown, both in magnitude and line of action. It is known from the first principles of statics, and shown graphically by the force polygon, that if all the forces in a balanced system, except two, are known, *any two which will close the polygon will hold the system in equilibrium*, provided that the resultant moment of the whole system about any axis normal to its plane (in the arch all forces may be taken to lie in one plane,) is equal to zero. From these considerations it at once follows that any one point in each of the joints $L\ M$ and $C\ G$ may be assumed as the centres of resistance for those joints, or, what is the same thing, as the points of application of the resultant forces acting on those joints. Let a' and a'', Fig. 6, be those assumed points.

The magnitudes and directions of the forces acting in $L\ M$ and $C\ G$, or through a' and a'', still remain to be found, and *the direction of one of them may be assumed.* The direction of the one acting through a'' will be assumed parallel to $B\ E$ and it will be called T_0; the line representing it is removed to one side, for the purpose of avoiding complication in the diagram. By taking the moments of T_0 and the resultant external forces acting on the ring stones about the point a', the magnitude of T_0 can at once be found. The remaining force, R, acting in the joint $L\ M$ becomes immediately determinate, both in direction and magnitude, and can be found by the use of the force polygon. These operations will presently be carried through in detail, by means of equations and diagrams.

The two forces, T_0 and R, which have just been determined, are but one of an indefinite number of pairs which may be used to complete the equilibrium of the portion $C\ L\ M\ G$. The remaining portion of the problem is to determine which of all these pairs will really exist in any given case. The complete solution of this question is perhaps impossible, but an approach to it, at least, may be made. The force or thrust T_0 is either partially or wholly a passive force called into action by the resultant external forces acting on the ring stones, which play the part of active forces.

Digitized by Google

By the principle of least resistance this passive force T_0 must be the least possible, consistent with the equilibrium of the structure.

The absolutely least value of T_0 consistent with stability can scarcely be found, because the operation by which it is sought is a tentative one. When once the point a' and direction of T_0 are assumed (which assumptions have been shown to be consistent with equilibrium), however approximately, the least value of T_0 is not difficult to fix.

In Fig. 6, Plate I, let g be the centre of gravity of the portion of the arch $A\ v''\ L\ M\ G$ and its load, and let W represent the weight which acts through that point, while l_1, as shown, is the horizontal distance of the line of action of W from the centre of resistance a'. Let h represent any of the forces h_1, h_2, h_3, etc. (for the sake of generality these are considered not to be parallel), and let l be the normal distance from the line of action of h to a_1; then the sum of all the moments of the forces h about a' will be Σhl. By taking moments, therefore, about a' there will result:

$$T_0 y_1 = Wl_1 + \Sigma hl \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore T_0 = \frac{Wl_1 + \Sigma hl}{y_1} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In Eq. (2) y_1 is the lever arm of the thrust T_0, or, the normal distance of its line of action from a', whatever may be the direction of T_0.

If the stability of position of any joint is secured, the centre of resistance must not depart farther from the centre of figure than a certain safe distance. Denoting the thickness, or depth of the springing joint $L\ M$ by t, the limiting distance of a' from the centre of $L\ M$ will be indicated by $q\ t$. The distance $M\ a'$ will be taken as $\frac{1}{2}t - q\ t = t(\frac{1}{2} - q)$. This assumption fixes the location of a', but leaves that of a'' movable within safe limits.

With the position of a' fixed, however much a'' may be moved in the joint $C\ G$, the lever arms l_1 and l will remain unchanged. Consequently, *the highest position possible of a'', which makes y_1, the greatest possible, makes, by Eq. 2, T_0 the least possible*, with a given set of values for the forces h. In fact, the form of Eq. 2 shows, in general, *that in order to have T_0 as small as possible, a'' should be as high as possible.* In general, also, a' should be as low (or as

Digitized by Google

near to M) as possible in the joint $M\,L$; but, since a movement of a' in some cases may increase l and decrease l_1, or *vice versa*, it may be necessary to locate that point in some position higher than its lowest in such cases.

Such, with a given set of values of the pressures h, is the general method of determining the most probable (*i. e.* least) value of the thrust T_0 by aid of the principle of least resistance.

It has been observed that the pressures h (usually conjugate to the vertical) have possible values lying between certain limits, and, in the general case, may have varying possible directions. Eq. 2 shows that, if other things remain the same, T_0 will decrease as Σhl decreases. Consequently, by the principle of least resistance, *the pressures, h, should be so taken that their magnitudes and lever arms may be the least possible; or, analytically, Σhl should be the least possible.*

The preceding methods and observations are perfectly general, and apply to all cases whatever; but if the characteristics of a given case are more simple, their applications become correspondingly easy.

It very frequently happens that the entire loading of an arch, if reduced to a uniform mass of the same density as that of the ring stones, may be taken as limited by a plane upper surface, $B\,E$, without sensible error. A *sloping* plane upper surface, like that in Fig. 6, will first be considered. With such a loading it frequently happens that the pressures h are conjugate to the vertical and parallel to $B\,E$, in which case their direction is fixed and cannot change however much the magnitudes or intensities may vary. Under these circumstances, with a given location of a', the lever arms l are invariable. In seeking the least value of T_0 by Eq. 2, therefore, the magnitudes h, alone, are to be taken as the least possible when y_1 and l_1 are fixed.

If the upper surface $B\,E$, Fig. 6, or conjugate direction, becomes horizontal, the forces h are horizontal forces, and the lever arms l are simply the vertical distances from the action lines of those forces to the point a'; and as before, in seeking T_0, Σhl is to be made as small as possible by decreasing h only. In this restricted case, if the springing joint $L\,M$ is horizontal, the lever arms l are absolutely invariable. Consequently, while, as before, a'' is to be taken as high as possible in the joint $C\,G$, the point a' is *always* to be placed as near as possible to M, because l_1 will then have its least value, and y_1 will be independ-

Digitized by Google

ent of the position of a'; the least values of h will then secure the least possible value of T_0.

Whatever may be the inclination of the springing joint $L\ M$, and whatever may be the character of the loading, if the forces h are all zero, the value of T_0 becomes:

$$T_0 = \frac{Wl_1}{y_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

As l_1 decreases on the approach of a' to M, and as y_1 increases as a'' departs from G, Eq. 3 shows that for the least value of T_0, in this case, *the centre of resistance at the springing joint must approach the soffit as near as possible, while in the joint C G that centre should be as far as possible from the soffit.*

This case would happen if the load were wholly vertical and applied to the ring stones at isolated points.

The preceding methods will give, in all cases, the most probable value for the thrust T_0; but there still remains the force R, or reaction, applied to the springing joint. It is to be found as follows, by the aid of the force polygon:

Fig. 7 represents the arch ring $L\ C\ N\ O\ G\ M$ of Fig. 6, with the external forces acting on each of the stones of which it is formed. The force 1 in Fig. 7 is the resultant of the external forces acting upon the stone $L\ M\ a$, Fig. 6; these external forces include h_1, and the weight of the stone itself, as has already been explained. In the same manner the forces 2, 3, etc., Fig. 7, are the resultant external forces acting upon the second, third, etc., stones from $L\ M$ respectively.

The upper part, $T_0\ 4\ 3\ 2\ 1$, of Fig. 8 represents these forces laid off in their proper directions, *i. e.*, around the polygon in the same way. The remaining side, R, of the polygon will give the direction and magnitude of the reaction R acting through a'. In the same manner the external forces acting on the stones between $C\ G$ and $O\ N$, combined with T_0, will give the other reaction R_1, *whose point of application, a_1, is not yet known.*

The line of resistance $a'\ a\ b\ c\ a''$ (a'' is at the intersection of the action line of T_0 and $C\ G$, Fig. 7, and is also shown in Fig. 6) can now be constructed by the ordinary method. In Fig. 8, P is at the intersection of R and R_1. From P radial lines are drawn to the different angles of the polygon as shown. In Fig. 7, the line of action of R is prolonged until it cuts that of

Digitized by Google

force 1 (also prolonged if necessary.) From the intersection of 1 and R the line 9, Fig. 7, is drawn parallel to 9 of Fig. 8 until it cuts the line of action of force 2. From this last intersection the line 10 is drawn parallel to the same figured line in Fig. 8 until it cuts the line of action of force 3. This simple operation is repeated and continued throughout the arch ring until from the intersection of 13 and 7, Fig. 7, a line (R_1) is drawn parallel to R_1 of Fig. 8. The intersection of this line with the joint $O\ N$ gives the point of application of the reaction R_1, *a point not before known.*

The intersection of the various joints (or the joints prolonged) with the lines R, 9, 10, 11, etc., Fig. 7 (or those lines prolonged), are the centres of resistance of those joints. Found continuously, they constitute the line of resistance for the arch ring. In order that the stability of position may be secured for each joint, its centre of resistance must be found within a certain distance of its centre of figure; this distance is frequently indicated by $q\ t$, and has already been used.

With the first trial value of T_0, it will frequently, and perhaps always, be found that the consequent construction will give some centres of resistance lying further from the centre of figure than $q\ t$. It will then be desirable to so change T_0 that the resulting line of resistance will, if possible, lie within the proper limits.

A consideration of Fig. 8, in connection with Fig. 7, will show, in a general way, in what manner a change in T_0 or the values of the pressures h, will influence the location of the centres of resistance.

If the pressures h remain the same while T_0 becomes smaller, the dotted lines in Fig. 8 will become steeper, consequently *all points in the line of resistance will rise relatively to the end points* a' *and* a_1. On the contrary, if, while the other circumstances remain the same, T_0 becomes larger, *all points will fall relatively to the points* a' *and* a_1. If the point a'' in $C\ G$ remains fixed while the same changes take place, a decrease in the value of T_0 will cause all the other centres of resistance to *fall*, while an increase will cause them to *rise*.

If the pressures h are conjugate to the vertical, *i. e.*, parallel to $B\ E$ in Fig. 6, it is not easy to predict, without reference to a particular case, what effect a uniform variation of those pressures will always produce, and much less so when they are not

Digitized by Google

conjugate. It may be stated as a general principle, however, that any variation whatever, either in the values of T_0 or h, which makes the lines radiating from P, Fig. 8, more inclined to a horizontal line will make the line of resistance more concave downward (or convex upward), while any variations in those quantities which cause the lines radiating from P to become less inclined will make the line of resistance less concave downward (or convex upward).

The preceding simple observations taken in connection with the circumstances of any particular case will enable the centres of resistance to be located within the safe limits, if possible, after one or two trials; and in the event of its not being possible, the necessity of increasing the depth of the arch ring will be shown.

It is proper to observe that the polygon R, 9, 10, 11, 12, etc., is not necessarily the polygon, or line, of resistance, though it is in most cases. That line is the polygon (or line) of pressures, and it is sometimes necessary to produce the sides of the polygon of pressures to the corresponding joints (as has before been observed) in order to find the centres of resistance.

In applying equations 1, 2, and 3, it is supposed that the lever arms l, l_1 and y_1 are measured from a large scale drawing; it is also supposed that the magnitude and position of the forces 1, 2, 3, 4, 5, etc., are found by construction in the same drawing.

There are thus found complete methods by which it can be determined whether or not a line of resistance can be constructed within the safe limits of any given arch ring, or by which an arch can be designed of such dimensions that a line of resistance may be found within safe limits. It then becomes a matter of importance to determine what these "safe limits" are. They can be definitely determined only by experience; but there are certain considerations of great weight which should be borne in mind in connection with the empirical aspect of the question.

Some authorities, as Rankine, considered the "safe limit" to be the middle third of the arch ring, in which case $q=\frac{1}{6}$; others make $q=\frac{1}{4}$, or the "safe limit," the middle half of the arch ring.

In all cases it is virtually assumed that the pressure exerted on any joint is one of uniformly varying intensity. While this law can not be demonstrated, it seems reasonable in the general case and it is the best working hypothesis that can be advanced.

When q is taken as $\frac{1}{6}$ it is made certain that no tension can

Digitized by Google

exist at any part of the joint. Fig. 10 represents this case.

All lines parallel to $A\ a$ represent *intensities* of stress; that line itself standing for the greatest intensity existing in the joint, while $A\ b = t$ is the joint, or rather a trace of its plane. The intensity at b is zero. The stress existing in $A\ b$ is simply a system of parallel forces whose centre is at R, $\frac{1}{3}$ of $A\ b$ from A, or $\frac{1}{6}\ t$ from the middle point of $A\ b$. If the centre of stress R never approaches nearer to A, pressure only can exist in the joint and it (the joint) will never tend to open. Thus, no portion of the joint will ever be subjected to tension, and since it is not safe to rely upon an ordinary masonry joint to resist tension, it is thought that q should have the limiting value of $\frac{1}{6}$.

It does not follow, however, that the joint will necessarily begin to open if R approaches nearer to A than $\frac{1}{3}\ t$, or, that the stability of the arch is necessarily endangered if the joint does begin to open.

In Fig. 11 the greatest intensity of stress is $A\ a$. At b the intensity is zero, while no stress at all acts on $B\ b$. R being the centre of stress the distance $A\ R$ is one-third of $A\ b$. Now, if the intensity $A\ a$ does not exceed the ultimate resistance to compression of the material there can be no opening at B, except that due to the elasticity of the material between b and A, and that is scarcely sufficient to bring the case within the meaning of "an open" joint. If the material at A is crushed, the joint truly begins to open; but even then the stability is not necessarily endangered. In that case the area representing the stress acting at the joint is $A\ d\ e\ b'$, $A\ d$ being the ultimate resistance to compression. The intensity is uniform in the vicinity of A, over an area indicated by $d\ e$, and the centre of pressure, or resistance, is nearer to A than R. Eventually, when the area of uniform intensity, equal to the ultimate resistance to compression indicated by $d\ e$, becomes too great, the whole ring stone will crush and the arch will fall, but not before. It is thus seen that the value of q may considerably exceed $\frac{1}{6}$, while the stability of the arch remains secured, even on the safe and proper supposition that the joint cannot resist tension.

If the joint were capable of exerting tensile stress, its condition would be given by Fig. 9. The triangle $b\ B\ c$ may represent tension while $A\ a\ c$ represents compression. Since, however, it is not prudent to rely upon the tensile resistance of a mortar joint let the Figs. 10 and 11 be again considered.

Digitized by Google

It certainly is dangerous to allow the greatest intensity of stress $A\ a$ to approach the ultimate resistance to compression of the material, and since the portion $b\ B$ of the joint in Fig. 11 is useless, the best method of procedure is to take $q = \frac{1}{6}$ and give the greatest intensity of stress, $A\ a$, a reasonable value. According to Rankine (Civil Engineering), in many arches which are considered first-class specimens of their kind the depth of key-stone is such that the ultimate resistance to crushing is from 3 to 70 times the greatest intensity of stress acting in it; while he remarks that the first value is too small and the latter too large. If the depth of arch ring is so designed that the greatest intensity of pressure, acting on any joint, under the heaviest loading is from 1-20 to 1-30 the ultimate resistance to crushing, ample security will be provided.

If the ultimate resistance per unit of area is represented by C, and the greatest intensity of working pressure allowable, by C', the following values will be used hereafter:

$$q = \tfrac{1}{6}, \text{ and } C' = \frac{C}{20} \text{ to } \frac{C}{30} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Since C' can exist at but one point in the section, the value of C may be taken as that belonging to large cubes, which is considerably larger than that belonging to small cubes.

In fixing the limits of the pressures h, no little difficulty arises in many cases. If the spandril filling is of dry or ideal earth, the limits are well defined; the lower is the conjugate pressure of earth loaded with its own weight, while the upper is simply the abutting power of the same earth. Such earth, however, is seldom or never found. Damp earth solidly packed, whose tenacity and compressive resistance is considerable, is the material usually found over the ring stones of an arch whose filling is not masonry. Such earth is capable of exerting a pressure against the ring stones, whose intensity will vary from 0 to some value higher than the abutting power of the same earth when dry and granular. It is usually impossible to determine the upper limit of this intensity by experiment, consequently the value to be taken must be a result of the judgment acting upon all the circumstances of the particular case. It should be a matter of care not to take too high a value, for such an error would involve a mistake on the side of danger in the question of stability; the value should be too small rather than too large.

Digitized by Google

If the spandril filling is of masonry, so far as the stability of the arch ring is concerned, the intensity of the pressure h may vary from zero to the working resistance to compression of the material of the spandril filling, if it is less than that of the arch ring. It is implied, however, that the values of h will not be of such magnitude as to give T_0 a value sufficiently large to cause the centre of resistance of the lowest joint in the abutment to move outside the safe limits. To illustrate this matter, let Fig. 5 represent half of an arch with its abutment. T_0 and the pressure acting against $D\ E$ are supposed to be horizontal in direction, but they are opposite to each other. Let their difference be represented by H. W is the weight of the mass, $B\ C\ E\ D$, acting through G (usually not the centre of gravity). The resultant force, therefore, acting on the half arch and abutment, will be R, and its line of action cuts the lowest joint, $D\ B$, at A, the centre of resistance. The pressures h must not be large enough to so increase the value of T_0, and hence H, that the point A will be found at a distance greater than $q\ t$ from the middle point of $D\ B$. This implicit condition is always to be borne in mind.

In designing an arch much time and trouble may be saved by an approximate rule for fixing a trial depth of key-stone, and such a rule may easily be established.

Let the heaviest moving load (covering the whole arch) and all the fixed load, including the weight of the ring stones, be reduced to a mass of the same density as that of the ring stones or voussoirs, in Fig. 5, and let the horizontal line $E\ C$ be supposed to limit the upper surface of such a mass. At the crown, F, the arch will be normally pressed by a load whose intensity may be represented by $K\ C = d$. Now the radius of the line of resistance (or linear arch) at the crown may not be very different from that of the intrados at the same point; hence they will be taken as equal and denoted by r, while w will represent the weight per cubic unit of the arch ring.

If, then, k is the depth of key-stone, there will result:*

$$T_0 = \frac{k\ C'}{2} = w\ d\ r \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$\therefore k = \frac{2\ w}{C'}\ d\ r \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

*The condition of stress exhibited by Fig. 10 is supposed to exist at the crown. Hence q will be equal to $\frac{1}{6}$.

Digitized by Google

This, of course, is only an approximate formula, and is limited by all the conditions assumed, but it furnishes an excellent first-trial value for the depth of key-stone.

If, for good sand-stone, $C' = 57600.0$ pounds (per square foot), and $w = 136$ pounds, there will result :

$$k = 0.005\ d\ r \text{ (nearly)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which d must be taken in feet. According as r is given in feet or inches, k will be found in feet or inches also. Ordinarily r is given in feet.

In any case of importance the value of C (hence also of C') should be determined by experiment with the stone to be used.

As with truss bridges so with arches, the moving load, per unit of span, may be decreased as the span increases. For this reason the quantity d has been assumed by some to vary inversely as the square root of the radius at the crown. Under such an assumption, if c is a constant, Eqs. 6 or 7 will take the form:

$$k = \frac{c}{\sqrt{r}} r = c\sqrt{r} = \sqrt{c^2 r} \ldots\ldots\ldots\ldots\ldots (8)$$

Rankine, in his "Civil Engineering," gives $c^2 = 0.12$ for a single arch and 0.17 for one in a series.

This assumption, however, does not seem to be warranted by the circumstances of actual loading. The moving load for arches cannot certainly be supposed to vary inversely as some function of the span any more rapidly than does that for truss-bridges. For the latter class the variation may very well be shown by taking a moving load per foot of span of 4,500 pounds for a 25 foot, and 2,500 pounds for a 500 foot span. If the law were expressed by the reciprocal of the square root of the span, the loads per foot would be 4,500 pounds and 1,000 pounds (nearly). Hence the moving load does not decrease nearly as rapidly as the law requires. Again, in general, the depth of fixed load at crown will *increase* as the span increases; this will tend to balance the *decrease* in the moving load.

Eq. 8, therefore, cannot express the law of variation of the thickness k with any very close degree of approximation. It should be observed, however, that, as Rankine shows, it does give very closely the actual depths of the key-stones of many arches that have stood well for many years. Such evidence,

Digitized by Google

however, is not conclusive unless it can be shown that there is neither excess nor deficiency of material in the arches taken as examples.

The considerations advanced in regard to the fixed and moving loads would seem to indicate that the thickness of key-stone, with ordinary conditions of loading, should vary about as the first power of the radius. In a great many existing examples k ranges from $\frac{r}{10}$ (for radius of 20 feet) to $\frac{r}{30}$ (for radius of 160 feet).

The simple matter of frictional stability yet remains to be noticed. No sliding of blocks upon each other can take place if the greatest obliquity of resultant pressure at any joint does not exceed the angle of repose. The obliquity of resultant pressure is at once obtainable after the true force polygon is known. Hence the means by which the stability of position is secured will also give all the information desired in reference to the stability of friction.

Application of the preceding methods will next be made to two examples.

EXAMPLE I.

The first example is the elliptic arch represented in Fig. 1 of the plate. The arch ring $M\ L\ C\ G$ is of the uniform thickness 5.4 feet. The principal dimensions are the following:

Half Span, $M\ N = 65.00$ feet.
The Rise, $N\ G = 32.50$ feet.
Thickness, $G\ C = L\ M = 5.40$ feet.
Depth of Loading, $A\ C = B\ D = 2.17$ feet.

The complete arch is semi-elliptic. Hence $M\ N$ and $N\ G$ are semi-transverse and conjugate axes, respectively. The radius of curvature at G will, then, be $(65)^2 \div 32.5 = 4 \times 32.5 = 130.00$ feet.

For convenience of calculation the weight of the arch ring and its loading (reduced to the same density) will be taken at 150 pounds per cubic foot. It will be assumed, for a purpose of comparison hereafter, that the upper surface of the total load (moving and fixed), reduced to the same density with the ring-stones, is the plane whose trace is $B\ A$. The figure $A\ B\ L\ M\ G\ A$ will then represent a mass of material of the uniform density of 150 pounds per cubic foot. The arch ring is divided

Digitized by Google

into eight parts by the joints shown at b, d, t, f, l, o and p. From the upper extremities of those joints vertical lines are drawn, $a_1\ a$, $c_1\ c$, etc., to $B\ A$, in such a manner that $B\ e_1$ is divided into three equal parts and $e_1\ A$ into five such parts.

$$B\ a_1 = a_1\ c_1 = \text{etc.} = 5.42 \text{ feet.}$$
$$e_1\ f_1 = f_1\ l_1 = \text{etc.} = 10.83 \text{ "}$$

The centres of gravity of the various six-sided figures comprised between the different pairs of vertical lines and corresponding joints, as shown at c', c'', c''', etc., were found by trial with a knife edge. The arch and loading were laid down, to a large scale, on heavy paper; the different portions, as $a_1\ c_1\ c\ d\ b\ a\ a_1$, were then cut out and carefully balanced on the knife edge. The centre of gravity of the entire half arch and load was, in the same manner, found to be at the point g. The areas of these portions were easily and accurately found from the same large scale drawing. Since the curve $M\ G$ is the quadrant of an ellipse the total area of the figure is

$$BA \times AG + LM \times GN + MN \times GN\left(1 - \frac{\pi}{4}\right) = 1163 \text{ square feet.}$$

These different areas multiplied by 150 will give the weights of the same portions. The various weights found in this manner, are given below and are represented in the figure by the *full portions* of the vertical lines drawn through the various centres c', c'', c''', etc.:

W_1=39300.00	pounds.	W_5=21045.00	pounds.
W_2=24370.00	"	W_6=16605.00	"
W_3=18705.00	"	W_7=13755.00	"
W_4=27765.00	"	W_8=12900.00	"

Total weight, W=174400.00 pounds.

The forces h are taken to be horizontal, and the spandril filling will be assumed to be wholly composed of earth, packed hard and close, such as is usually found under similar circumstances; no tympan of masonry will be supposed. The intensity of each horizontal force will be assumed uniform, consequently its centre will be at the centre of its surface of action; no error on the side of danger is committed by this assumption. Loose earth would be able to exert a pressure against the arch

Digitized by Google

ring equal to its abutting power; the same earth packed hard, with the aid of the little dampness which it naturally absorbs, will be capable of exerting a much greater pressure. The abutting power, however, will be taken as the upper limit. The lower limit, in such a case, may be zero.

The earth in its loose state will be supposed to weigh one hundred pounds per cubic foot, and to have an angle of repose equal to thirty degrees; or, $w=100.00$ lbs. and $\theta=30°$. The horizontal force h_1 will act on $n\ L$, h_2 on $n'\ a$, h_3 on $n''\ c$, etc. *The intensity in loose earth, which would exist at a point about one-quarter the vertical height of any one of those surfaces below its top, will be taken as the uniform intensity for the surface in question.** In the general case this is the same as taking the intensity at the middle point of the surface pressed, considering the earth as of the same density with the actual loading. The selection of such a point will also tend to a decrease in the values of h, and will consequently be the origin of an error on the safe side.

The formula for the abutting intensity of a loose mass of earth with horizontal upper surface and at the depth x is:

$$p = wx\frac{1+\sin\theta}{1-\sin\theta}$$

In this case $\sin\theta = 0.5$. Inasmuch as the density of the masonry and load has been taken at 150.00 lbs., and that of the loose earth at 100.00 lbs., it will be necessary to increase the actual depths of loading by one-half in order to obtain a value of x in the determination of p. The different values of x found in this manner for $B\ L$, $a_1\ a$, $c_1\ c$, etc., are:

$$\begin{aligned}
x &= 40.07 \times \tfrac{3}{2} - 12.1 = 48.00 \text{ (nearly).}\\
x &= 24.30 \times \tfrac{3}{2} - 4.5 = 32.0 \quad \text{“}\\
x &= 18.2 \times \tfrac{3}{2} - 3.0 = 24.3\\
x &= 14.3 \times \tfrac{3}{2} - 4.05 = 17.4\\
x &= 9.0 \times \tfrac{3}{2} - 2.60 = 11.0 \text{ (nearly).}\\
x &= 5.6 \times \tfrac{3}{2} - 1.40 = 7.0
\end{aligned}$$

*This is an unnecessary refinement, as a few practical examples will demonstrate. It is always sufficient to take an intensity at the middle of the actual surface pressed. In such a procedure the negative quantities in the following values of x would be two-thirds of those given. As, however, the diagrams have been drawn for the text as originally written, and as the results would not be sensibly different, and finally, as the operations would be identical, neither text nor plate have been changed in this third edition. W. H. B.

Digitized by Google

The very small values of h for the portions of 7 and 8 of the arch ring will be neglected.

The horizontal forces h will be at once found, using those values of x, by multiplying $p = wx \times \frac{1 + \sin\theta}{1 - \sin\theta}$ into the area over which it acts. These areas are $\frac{4}{3}$ the negative quantities found in the second members of the values of x. Beginning with the surface $L\,n$ there will result:

$$h_1 = 14400.00 \times \tfrac{4}{3} \times 12.15 = 233280.0 \text{ pounds.}$$
$$h_2 = 9450.0 \times \tfrac{4}{3} \times 4.5 = 56700.0 \text{ "}$$
$$h_3 = 7290.0 \times \tfrac{4}{3} \times 3.0 = 29160.0 \text{ "}$$
$$h_4 = 5220.0 \times \tfrac{4}{3} \times 4.05 = 28200.0 \text{ "}$$
$$h_5 = 3300.0 \times \tfrac{4}{3} \times 2.6 = 11550.0 \text{ "}$$
$$h_6 = 2100.0 \times \tfrac{4}{3} \times 1.4 = 4200.0 \text{ " (nearly).}$$

The vertical distances of the centres of the surfaces $L\,n$, $a\,n'$, $c\,n''$, etc., above the horizontal joint $L\,M$, measured from the large scale drawing, are as follows, l_1' belonging to $L\,n$, l_2 to $a\,n'$, etc.:

$l_1' = 8.1$ feet. $\quad l_4 = 28.7$ feet.
$l_2 = 19.1$ " $\quad l_5 = 33.0$ "
$l_3 = 24.1$ " $\quad l_6 = 35.8$ "

In accordance with the principles which have been already stated, the centre of resistance, a', of the joint $L\,M$ is taken one-third of $L\,M$ from M; also a'' is one-third of $C\,G$ from C. Consequently the value of y_1, or the height of a'' above a', is 36.1 feet. T_0 is taken to be horizontal.

Taking the sum of the products $h_1\,l_1'$, $h_2\,l_2$ etc., there will result the numerical value of $\Sigma\,h\,l$. The horizontal distance of the centre of gravity g from a' is $l_1 = 20.8$ feet; this is the l_1 of the product $W\,l_1$ in Eq. 2. Hence $W\,l_1 = 3627520.0$. Putting all these numerical values in Eq. 2,

$$T_0 = \frac{8652340.0}{36.1} = 240000.00 \text{ pounds (nearly).}$$

In Fig. 2 is represented the force polygon drawn to a scale of 100000.00 pounds per inch. The forces 8 and 7 are simply W_8 and W_7. The force 6 is the resultant of W_6 and h_6 as shown in Fig. 1. The force 5 is the resultant of W_5 and h_5;

Digitized by Google

4, of W_4 and h_4; 3, of W_3 and h_3; 2, of W_2 and h_2; 1′, of W_1 and h_1. They are all represented in Fig. 1. The line extending from f to e represents T_0. The dotted line drawn from d to e gives the reaction at a', Fig. 1. From a' a line $a'k$ is drawn until it intersects the force 1′ prolonged; from k a dotted line is drawn parallel to one (not shown) extending from e to the lower extremity of 2 (Fig. 2) until it cuts the prolongation of the line of action of force 2 in Fig. 1. The remainder of the "equivalent polygonal frame," shown by the broken line $a'k\,e\,a''$, is drawn in the same manner and as usual. Inasmuch as a large number of centres of resistance (those belonging to the joints $a\,b$, $c\,d$ and $e\,t$) lie outside the arch ring, another value of T_0 and corresponding frame, must be determined if the equilibrium of the arch is to be established.

Let it be supposed that h_1 is zero; there will then result:

$$T_0 = \frac{8652340 - 1889568}{36.1} = 187700.00 \text{ pounds.}$$

$c\,f$, Fig. 2, represents this value of T_0, and W_1 takes the place of 1′, while $d'c$ is the reaction at a'. The equivalent polygonal frame corresponding to this value of T_0 is not shown. All centres of resistance lie within the arch ring, however, but too near the soffit in the joints $a\,b$ and $c\,d$.

The next value of h_1 was taken at 44570.00 pounds. It gave $T_0 = 197700.$ pounds, which is represented by $f\,b$ in Fig. 2. In the same figure $b\,d''$ is the corresponding reaction R. The resulting polygonal frame (not shown) still gives the centre of resistance in $a\,b$ a little too near b.

Again, a value of 66885.00 pounds was assumed for h_1, and the resulting value of T_0 was found to be

$$T_0 = 187700.00 + \frac{541500.00}{36.1} = 202700.00 \text{ pounds.}$$

This is represented in Fig. 2 by $a\,f$, and 1 represents the resultant force acting on the stone $L\,M\,a\,b$. The complete force polygon is, then $a\,f$, 8, 7, 6, 5, 4, 3, 2, 1, R; the latter representing the reaction at a'. The radial lines are drawn as shown in Fig. 2, from a to the other angles of the polygon. The sides of the polygonal frame shown by the full lines within the arch

Digitized by Google

ring, Fig. 1, are parallel to these radiating lines and drawn in the usual manner. Through a' a line is drawn parallel to R, until it cuts force 1, from that point of intersection a line is drawn parallel to p until it cuts force 2, from that point of intersection, a line parallel to p' is drawn until it cuts force 3, and so on until the point a'' is reached. The intersections of the sides of this polygonal frame with the corresponding joints all lie within the middle third of the arch ring; consequently the arch is stable. This line of resistance *may not be the true one for the given load, but it is demonstrated that the middle third of the arch ring contains at least one line of resistance; stability of position, therefore, is secured.*

This is the same arch as that treated by Scheffler in § 26 of the French translation of his work. He found it impossible to construct a line of resistance within the *arch ring*, and much less within the middle third. He ignored the forces h, as his method does not contemplate their existence. The example was intentionally taken to illustrate the important influence which conjugate forces exert on the stability of an arch ring.

It is to be noticed with what ease the line of resistance is made to move in any desired direction after the first trial is made.

The greatest obliquity of pressure is only 11° 30′ on the joint $a\,b$, a value far less than the angle of repose. Hence frictional stability is secured.

The depth of loading $A\,G$ (Fig. 1) is 7.57 feet. The radius of curvature at A has already been found to be 130.00 feet. Hence, for the particular suppositions on which it is based, Eq. 7 would give for the thickness of the key-stone:

$$k = 0.005 \times 7.6 \times 130.00 = 4.94 \text{ feet.}$$

The greatest intensity of pressure at the crown is easily determined by the assumptions made in Fig. 10. If C' is the intensity desired, there results:

$$C' \times \frac{5.4}{2} = 202700.00 \therefore C' = 75000.00 \text{ pounds (nearly).}$$

This is the intensity per square foot, and is within the safe limit of nearly all good building stone. Fig. 2 shows that no

Digitized by Google

joint is subjected to a greater resultant pressure than T_0; hence 75000.00 pounds per square foot is the greatest intensity existing in the arch ring.

EXAMPLE II.

The second example is an elliptic arch precisely like the first, so far as the arch ring is concerned, but the assumed load is unsymmetrical. Fig. 3 represents the arch and its loading. The half of the arch $T K$ is supposed to carry a load whose uniform depth, when reduced to the density of the ring stones, is $A T = K H$. The other half carries a load of the same density, whose uniform depth is $K K' = \frac{1}{3} K H$.

As in Ex. I, the density of the ring stones is taken at 150 pounds per cubic foot.

The backing, $T K e L T$, is supposed to be earth, packed close and hard, whose density is 100 pounds per cubic foot. This mass of earth, reduced to the same density as that of the ring stones, is shown by the curved figure $t t' K e L t$, in which $L t = \frac{2}{3} L T$, $a t' = \frac{2}{3} a S$, etc.

The curved figure, $t t'' t''' H K t' t$, represents the uniform load $A H K T$ in such a manner that $t t'' = t' t''' = H K = T A$.

The following complete the data required:

Half Span $M N = 65.00$ feet.
Rise $N U = 32.50$ feet.
Thickness $K U = L M = 5.40$ feet.
Depth of Loading $H K = 3.00$ feet.
" " $K' K = 1.50$ feet.
$K O = 26.6$ feet. $O P = 14.4$ feet.
$P Q = 13.2$ " $Q S = 11.5$ "
$S T = 4.7$ feet.

The possible intensities of pressure between the arch ring and backing are assumed to lie between 0 and the abutting power of the earth. The joints $a b$, $e d$, $f g$, $h k$ are found by dropping verticals from the points $B D E F$. The half arch and load, $L t'' t''' H U M L$, is then drawn to a large scale on stiff paper, and its centre of gravity G is found, by balancing on an edge, to be horizontally distant from M, 21.2 feet, making $l_1 = 23.00$ feet, as will afterwards be seen. The stiff paper model is then

Digitized by Google

cut into the five portions shown, and the centres of gravity of the different portions are found, in the same manner, to be at the points c, c', c'', c''', c^{iv}. The areas and weights (areas multiplied by 150) represented by those areas are given below, commencing with $L\,t''\,t'''\,a\,b\,M\,L$:

$A_1 = 181.12$ sq. feet.	$W_1 = 27168.00$ pounds.
$A_2 = 251.22$ "	$W_2 = 37683.00$ "
$A_3 = 203.40$ "	$W_3 = 30510.00$ "
$A_4 = 166.50$ "	$W_4 = 24975.00$ "
$A_5 = 244.72$ "	$W_5 = 36708.00$ "
	$W = 157044.00$ pounds.

The abutting intensity of the earth will be taken at the middle points of the vertical projections of the portions $L\,a$, $a\,e$, $e\,f$, $f\,h$, $h\,K$. As in Ex. I, it will be assumed that these intensities are uniform over those vertical projections, whose heights (beginning with that of $L\,a$) are:

$a_1 = 13.6$ feet. $\qquad a_2 = 10.7$ feet.
$a_3 = 6.6$ " $\qquad a_4 = 4.2$ "
$a_5 = 2.5$ feet.

The centres (h', h'', etc.) of these areas are above the horizontal joint $L\,M$ by the distances:

$l'_1 = 6.8$ feet. $\qquad l_2 = 19.3$ feet.
$l_3 = 27.9$ " $\qquad l_4 = 33.3$ "
$l_5 = 36.7$ feet.

The values of x to be used in the formula $p = wx\dfrac{1 + \sin\theta}{1 - \sin\theta}$ are to be found by putting $x_1 = (37.9 + \frac{3}{2}\,3) - l'_1 = 35.6$; $x_2 = (37.9 + \frac{3}{2}\,3) - l_2 = 23.1$, etc. As before, θ will be taken at 30°. The horizontal forces acting against the various portions of the arch ring, will then be:

$$h_1 = p_1\,a_1,\ h_2 = p_2\,a_2,\ h_3 = p_3\,a_3, \text{ etc.}$$

In accordance with the principles already established, $M\,a'$ in Fig. 3 will be taken as $5.4 \times \frac{1}{3} = 1.8$ feet. By the general principles of the construction of equivalent polygonal frames, it is known that a frame for a large vertical load will, in general, rise higher than one designed for a smaller load. Hence the direction of T_0 at the crown *will not be taken horizontal, but*

Digitized by Google

inclined as shown in Fig. 3. For the same reason KC will be taken greater than one-third of KU. The inclination of T_0 from a horizontal line makes its lever arm y_1 greater for one side of the arch and less for the other, than would be found if it (T_0) were horizontal. *The inclination should be such that the greater lever arm will be found on that side of the arch carrying the greater load.* The distance CU was fixed at 3.1 feet, while the angle VCT_0 was fixed at 1° 30′. A trial angle of 5° was first taken but found to be too large. The lever arm y_1 is, therefore,

$$y_1 = 35.6 \times \cos 1° 30' + 66.8 \times \sin 1° 30' = 37.4 \text{ feet.}$$

The values of $p_1 a_1$ and $p_2 a_2$, as determined by the formula, were found to be about 145,000 pounds and 74,000 pounds, respectively. After two or three trials, however, the values $p_1 a_1 =$ 105400.00 and $p_2 a_2 =$ 43750 were found to give proper positions for the various centres of resistance; those values were therefore taken.

Making all these numerical substitutions, then, in Eq. 2 there will result:

$$T_0 = \frac{6522970}{37.4} = 174400.00 \text{ pounds.}$$

This is represented in Fig. 4, by the line T_0 drawn to a scale of about 68,000.00 pounds per inch.

In Fig. 3 the forces W_1 and h_1, W_2 and h_2 etc., are drawn in their proper positions in the usual manner. In Fig. 4 the various resultant forces 5, 4, 3, 2, 1 are so drawn as to form a polygon; consequently R will represent the reaction at a' in Fig. 3.

The radial lines drawn from I and indicated by p, p', p'', etc., are parallel to the sides of the polygonal frame shown in Fig. 3, which, by their intersections with corresponding joints, fix the centres of resistance a'', a''', a^{IV}, a^{V}.

The details of the two or three approximations by which the above value of T_0 was found, have been omitted, as they involve precisely the same operations, and those only, which were outlined in Example I.

By varying the magnitudes of the forces h, or the inclination of T_0, or the position of the point C, or all of them simultaneously, it is manifest that centres of resistance may be found which will lie nearer the centres of the joints than those shown

Digitized by Google

in Fig. 3. Again, it is to be remembered that the actual pressure of packed earth against the arch ring may be very much greater than the abutting intensity of the same earth in a dry granular condition.

These operations determine the stability of that half of the arch carrying the heaviest load. The other half must be treated in the same manner in order to fix the stability of the whole arch ring. This is not done, however, as the method has been sufficiently exemplified.

Fig. 4 shows that T_0 is the greatest pressure acting on any joint. Taking this in connection with the corresponding results of Ex. I, it is seen that no portion of the material of the arch ring will sustain an intensity of stress greater than the safe working limit.

Fig. 3, also, shows that the obliquity of resultant pressure on any joint cannot exceed the angle of repose. Hence stability of friction is secured.

In applying this method the largest and most accurate drawings possible must be made, and in most cases a greater number of joints must be taken than are shown in Fig. 3, or possibly, even, in Fig. 1. The scales to which the drawings (from which the figures in the plate were taken) were made, were ten feet, and fifty thousand pounds per inch. Such drawings are sufficiently correct for the present purpose, but would fail to give that degree of accuracy which should in all cases be reached in engineering experience.

It is to be noticed, however, that the method is applicable to any shaped arch or any kind of loading, in other words, it is perfectly general.

EMPIRICAL FORMULÆ FOR ARCHES.

The following empirical formulæ for arches have been proposed by different authorities and will serve at least to fix trial depths of key-stones or for approximate estimates:

Let v = rise or vers-sine of arch in feet.

" s = span of arch in feet.

" k = depth or thickness in feet of the arch ring at crown.

" R = radius of the intrados in feet, at crown.

" r = radius in feet of the circle which will touch the intrados at crown and the two springing joints, the latter being at the same elevation.

Digitized by Google

Let h = height or depth of earth loading at the crown, in feet.
" H = height of abutment, *i. e.*, from springing line to top of foundation, in feet.
" E = thickness of abutment in feet.

Then there has been proposed :—

$$E = 1 + 0.04\ (5\ s + 4\ H) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Perronet's rule is at follows :

$$k = 1 + 0.035\ s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Dejardin has proposed the following rules:

Semi-circular arch ; $e = 1 + 0.1\ R$ ..... (11)

Segmented Arches.

$$\frac{v}{s} = \tfrac{1}{6}\ ;\quad k = 1 + 0.05\ R \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\frac{v}{s} = \tfrac{1}{8}\ ;\quad k = 1 + 0.035\ R \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\frac{v}{s} = \tfrac{1}{10}\ ;\quad k = 1 + 0.02\ R \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Elliptical, false elliptical or basket-handled arches :

$$\frac{v}{s} = \tfrac{1}{3}\ ;\quad k = 1 + 0.07\ R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

If it is necessary to take into account the height of the earth loading at the crown ;

$$k = 1 + 0.035\ s + 0.02\ h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

It has been found that if the vertical projection of each and every joint is equal to the depth at the crown (*i. e.*, equal to e) the arch will usually be stable. This rule is to be applied only to those joints between the crown and that one whose inclination to a vertical plane through the axis of the arch is 60 degrees.

Trautwine's rule is :—

For first class cut stone :

$$k = \frac{\sqrt{r + \frac{1}{2}s}}{4} + 0.2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

For second class work :

$$k = \frac{9}{8}\left(\frac{\sqrt{r + \frac{1}{2}s}}{4} + 0.2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

For brick or fine rubble :

$$k = \frac{4}{3}\left(\frac{\sqrt{r + \frac{1}{2}s}}{4} + 0.2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Digitized by Google

NOTES ON THE

Measurement of Railway Earth-work.

WILLIAM G. RAYMOND.

It is not with the idea of presenting anything new to the older members of the Rensselaer Society of Engineers that these notes are published in this form.

The matter here given forms the theoretical portion of the instruction given in the Rensselaer Polytechnic Institute, in measurement of railway earth-work, and students are required to recite on these notes.

Their publication in this form gives them to the students at a minimum cost, and the Rensselaer Society of Engineers having kindly offered to print them, the writer accepts the offer with pleasure for the reason stated.

Should any of the younger practitioners find a suggestion in the notes the writer will be much gratified. W. G. R.

PRELIMINARY CONSIDERATIONS.

Railway earth-work is paid for by the cubic yard. All measurements of earth-work are based on the prismoid. The definition of a prismoid or prismatoid is "a solid having two parallel polygonal bases connected by triangular faces." This is equivalent, if the faces are conceived small enough, to the following, which somewhat better defines the forms dealt with in earth-work; a prismoid is a solid having parallel plain ends with sides formed by moving a ruled line around the perimeters of the parallel plain ends as directrices.

Such a solid may be conceived as made up of a combination of prisms, pyramids and wedges.

By Geometry the volume of a right

$$\left.\begin{matrix}\text{prism}\\ \text{wedge}\\ \text{pyramid}\end{matrix}\right\} \text{ is } V = \begin{cases} Ah \\ A\dfrac{h}{2} \\ A\dfrac{h}{3} \end{cases}$$

where A is the area of the base and h is the altitude of the solid.

The volume of any doubly truncated prism is $V = Ah$ in which A is the area of a right section and h is the element of length through the center of gravity of the right section or is the length of the line joining the centers of gravity of the two ends. (Johnson's Surveying, pp. 411.)

Digitized by Google

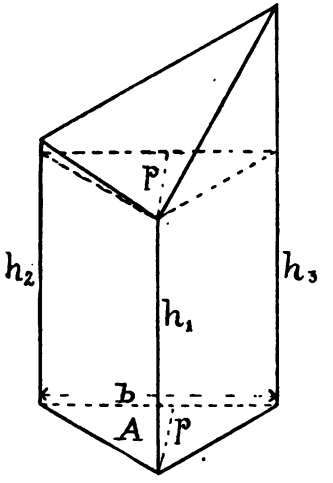


Fig. 1

In any prism having a symmetrical right base, the center of gravity of that base is at its center of form. This is true of all the regular polygons, and of all parallelograms.

In such prisms the length of the element through the center of gravity is the mean of all the edges. This is further true of a triangular prism.

This may be shown as follows:

In the truncated triangular prism shown in Fig. 1, the area of the right base is $A = \frac{1}{2} b p$ and $p = \frac{2A}{b}$.

The volume is $V = \frac{(h_2 - h_1) + (h_3 - h_1)}{2} b \frac{p}{3} + \frac{3h_1 A}{3}$. Substituting for p its value $\frac{2A}{b}$, $V = \frac{h_1 + h_2 + h_3}{3} A$.*

A truncated parallelopiped may be divided into two truncated triangular prisms in two ways as shown in Fig. 2, and each set treated as above and the resulting volumes added. This gives twice the volume of the solid, which divided by two gives $V = \frac{h_1 + h_2 + h_3 + h_4}{4} A$.

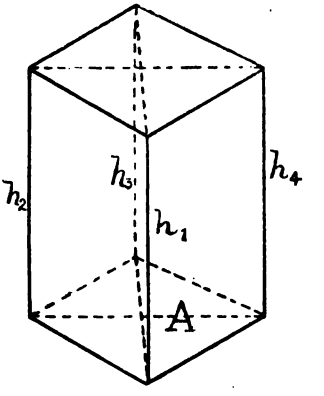


Fig. 2

A prism having for its right base a regular pentagon may be divided into three triangular truncated prisms in five ways and treated as the parallelopiped when the result $V = \frac{h_1 + h_2 + h_3 + h_4 + h_5}{5} A$ is obtained.

---

*The student may show that $\frac{h_1 + h_2 + h_3}{3}$ is the length of the element through the centre of gravity of the bases.

Digitized by Google

If a single expression can be found giving the volume of a prism, pyramid and wedge, such an expression will give the volume of a prismoid, or prismatoid, since such a solid is made up of the above three elementary solids, all having, in any given case, equal heights.

In the

$$\left.\begin{matrix}\text{prism}\\ \text{pyramid}\\ \text{wedge}\end{matrix}\right\} V = \left\{\begin{matrix}Ah\\ \frac{Ah}{3}\\ \frac{Ah}{2}\end{matrix}\right\} \text{ which may be written } \left\{\begin{matrix}\frac{h}{6}(A_1 + A_2 + 4A_m)\\ \frac{h}{6}(A_1 + A_2 + 4A_m)\\ \frac{h}{6}(A_1 + A_2 + 4A_m)\end{matrix}\right.$$

in which A_1 is one base, A_2 is the other base, and A_m is a section midway between the two end bases and parallel to them.

The single expression thus found is called the prismoidal formula and is the correct formula for finding the volumes of prismoids. *A_m is not a mean of A_1 and A_2, but each of its linear dimensions is a mean of the corresponding linear dimensions of A_1 and A_2.*

This formula is applicable to the sphere and all the regular solids of revolution.*

To avoid the labor involved in computing the volume of prismoids by the prismoidal formula one of two assumptions is frequently made.

These two assumptions are:

1. The volume of a prismoid is $\frac{A_1 + A_2}{2}h$, called method of average end areas.

2. The volume of a prismoid is hA_m, called method of mean areas.

The error of the first assumption is twice that of the second and of the opposite sign. To show this let a_1, a_2 and a_m be

* Let the student show the truth of this statement for the sphere and both spheroids of revolution.

Digitized by Google

be homologous sides of the similar end and middle sections of a given prismoid. Then since the areas of similar figures are as the squares of their homologous sides there may be written the following expressions for the volume of the prismoid based on the true and approximate formulas. Remembering that $a_m = \frac{a_1 + a_2}{2}$,

By prismoidal formula

$$V_p = 2K\frac{h}{6}(a_1^2 + a_1 a_2 + a_2^2) = \frac{4}{12} K h (a_1^2 + a_1 a_2 + a_2^2) \ldots\ldots (1.)$$

By average end areas

$$V_{ea} = \frac{Kh}{2}(a_1^2 + a_2^2) = \frac{6}{12} K h (a_1^2 + a_2^2) \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

By mean areas

$$V_m = K\frac{h}{4}(a_1^2 + 2a_1 a_2 + a_2^2) = \frac{3}{12} K h (a_1^2 + 2a_1 a_2 + a_2^2) \ldots (3.)$$

Subtracting (1) from (2)

$$V_{ea} - V_p = -\frac{2Kh}{12}(a_1 - a_2)^2 \ldots\ldots\ldots\ldots\ldots\ldots (4.)$$

Subtracting (1) from (3)

$$V_m - V_p = -\frac{Kh}{12}(a_1 - a_2)^2 \ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

It is seen that Eq. (5) is half of Eq. (4) and of opposite sign. From Equations (4) and (5) it is also seen that the average end area method gives results too great while the mean area method gives results too small.

By the prismoidal formula or one of the approximations mentioned the volume of all masonry work or earth-work may be computed.

It is seen from Equations (4) and (5) that the errors of the assumptions given vary with the square of the difference in dimensions of the two end areas, and therefore it may be concluded that in cheap work, such as earth-work, if successive end areas are nearly alike the simpler method of average end areas

Digitized by Google

may be used, while if the areas are quite different the prismoidal formula should be used; and in costly work such as masonry the prismoidal formula should always be used.

CROSS SECTIONING.

The usual cross-sectional form of a railroad cutting is one of the two shown in Figs. 3 and 4.

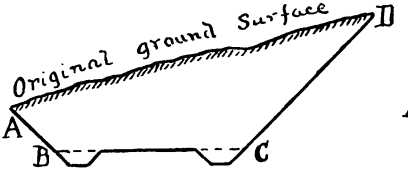


Fig. 3

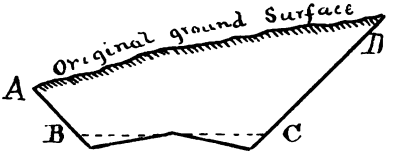


Fig. 4

On the road-beds shown is placed ballast and in the ballast the ties.

The dimensions of cross sections are determined at each station and oftener as may be deemed necessary. The area determined is *A B C D* leaving the excess for ditches, which is a uniform quantity per station, to be added in a lump for the entire length of each cut.

The slopes of the sides of the cutting depend on the material through which the cutting is made. The following are common:

Solid Rock, vertical or $\frac{1}{4}$ horizontal to 1 vertical.

Loose Rock, $\frac{1}{4}$ horizontal to 1 vertical or $\frac{1}{2}$ to 1 or $\frac{3}{4}$ to 1.

Compact Clay or Earth, 1 horizontal to 1 vertical at first; but the sides will not stand long at this slope and in some cases the slopes are made $1\frac{1}{2}$ to 1, and this is considered the best practice when money is plenty.

Loose Earth or Sand, from $1\frac{1}{2}$ to 1 to 3 to 1.

In embankment, earth slopes are usually made $1\frac{1}{2}$ to 1 and with light materials sometimes flatter. Rock slopes may be

Digitized by Google

made somewhat steeper. There are two general classes of railroad cutting "thorough cut" Fig. 5 and "side-hill" cut Fig. 6.

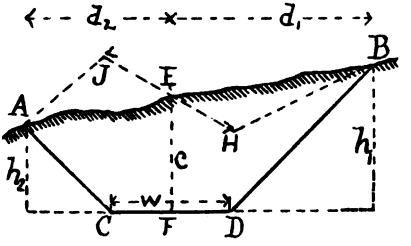


Fig. 5

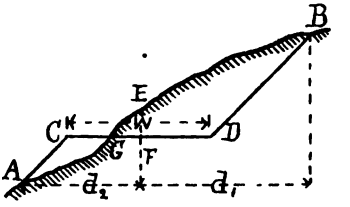


Fig. 6

Since embankments may be treated, so far as laying out and computing are concerned, as inverted cutting, the method of cross sectioning and determining areas and volumes will be given for cutting alone.

Cross sectioning consists in finding the points A and B where the surface is intersected by the planes of the sides and placing stakes at these points on which stakes are marked the station and vertical distance above the bottom of the cut or "grade" with the letter "C" to indicate cut. In fill the stake is marked "F," with the distance of the point below grade. There is also determined the "center cut" EF and in the case of side-hill work the "grade point" or point where the surface intersects the road-bed.

On the center stake is always marked the cut or fill and at the grade point is sometimes driven a "grade plug" the top of which is driven to be "at grade" and then chalked with keel or marking chalk.

In the note book are recorded the heights above or below grade of all the points in the cross section where stakes are driven or where changes in slope occur, with the distances of these points from the center *i. e.*, d_1, d_2, GF, &c.

The work is all done to the nearest one-tenth of a foot.

The method of determining the points A, B, and G is as follows:—

Digitized by Google

From the profile of the center line, on which has been plotted the grade line, is known the elevation of grade for each station.

From the general instructions of the Chief Engineer of the road and the nature of the ground is known the slope of the sides of the cut and the width of the road-bed $C\,D$.

From the nearest bench mark, the height of a level, so set as to command the station or stations whose section or sections may be desired, is determined and a rod then read on the center line at the required station as at E.

The difference between the elevation of the station thus found and the "grade" is the cut or fill at that point. (It is better to carry the height of instrument *above grade* rather than above the datum to which the levels have been referred; the difference between rod reading and instrument giving at once the cut or fill.)

To find the point B, it is evident that if s represent the slope of the side $= \frac{hor.}{vert.}$ then $d_1 = \frac{w}{2} + s h_1$. The rodman knowing the center cut estimates the value of h_1 and then mentally computes d_1 and holds his rod at that distance from E. The leveller reads it and sees if it gives the same value of h_1 that has been estimated by the rodman. If it does, the stake is driven and marked and the process repeated on the other side of the center. If h_1 is found to differ from the rodman's estimate he knows at once whether he is too far out or not far enough according as h_1 proves larger or smaller than his estimate and he tries again. An expert rodman will not—in ordinary ground—make more than one change, and frequently places his rod right at the first trial. The rodman is usually assisted by an axman, or an axman and tapeman, and sometimes the engineer carries the rod and keeps the notes, the rodman merely manipulating the level. Sometimes the work is done with a cross section rod from level notes of the center line previously taken. The writer prefers the method given. Convenient or necessary modifications will suggest themselves to the practicing engineer.

In case the ground be irregular as shown by the dotted line $B\,H\,J\,A$ the heights H and J above grade, together with their distances from E will also be determined and recorded.

Digitized by Google

When it is necessary to take only the center and side heights to correctly show the surface the cross section is called a "three level" section.

If h_1, C and h_2 are equal the section is called a "level section." There are many forms for keeping notes. The following is suggested, the direction of the diagonal line between h and d indicating either cut or fill. The sign — for cut and + for fill may be used or the signs may be reversed so long as the same system is used continuously.

Sta.	Center Line. Elev.	Center Line. Grade.	Left.	C.	Right.
18	564.2	560.0	h_2 / d_2	4.2	h_1 / d_1
19	568.4	561.0	h_2 / d_2 8.6 / 12.0	7.4	6.0 / 11.0 h_1 / d_1
86	564.2	570.0	h_2 / d_2	5.8	h_1 / d_1
			&c.		
102	572.4	570.0	h_2 / d_2 0.0 / 3.0	2.4	h_1 / d_1

The above represents the left page; the right page is used for remarks or for computations to be shown hereafter.

Station 18 is in cut and Station 86 is in fill.

The notes opposite station 19 show the form for the irregular section in Fig. 5 and those opposite Station 102 represent the form for Fig. 6. Instead of drawing inclined lines to represent cut or fill, horizontal lines may be drawn and the kind of work indicated by signs. Again the h's and d's may be inverted.

The kind of material of the cut, the slope, and width of road-bed may be written along the station column. It is usual to make the road-bed in cut 18 feet to 20 feet wide and in fill 12 feet to 16 feet wide.

Digitized by Google

At the place where a fill and a cut join, a surface grade line runs across the road-boad and usually diagonally. A plan of such a junction with the resulting cross sections is shown in Figs. 7 and 8.

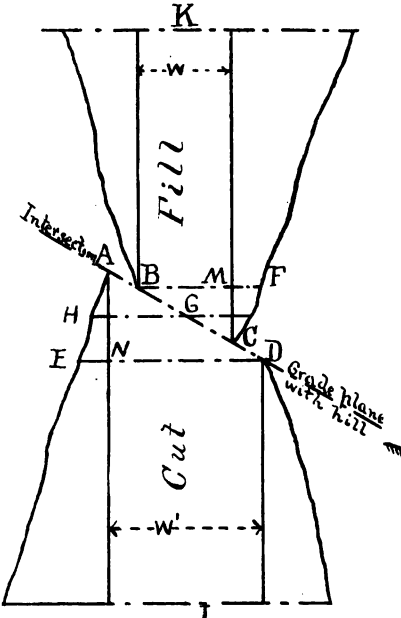


Fig. 7

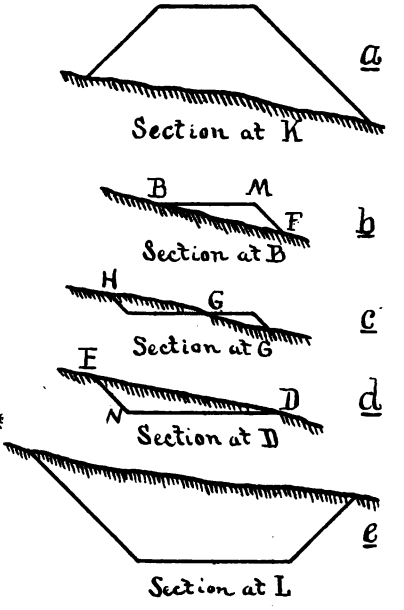


Fig. 8

To properly get the volumes the following data are wanted.

In fill—

Section at *K*, next station to *B F*.

Section at *B F*. (Omit the cut area here.)

The distance *M C*.

In cut—

The distance *A N*.

Section at *E D*.

Section at *L*, next station to *E D*.

The point *G* is usually found and a grade plug driven but it is not needed in computing.

Digitized by Google

The notes for these cross sections might be as follows. Assuming a 14 foot road-bed in fill and a 20 foot road-bed in cut. Slopes all $1\frac{1}{2}$ to 1.

Sta.	Center Line. Elev.	Center Line. Grade.	L.	C.	R.
21	462.2	465.0	3.4 / 12.1	2.8	2.0 \ 10.0
+ 40		464.6			0.0 / 10.0
+ 50	463.5	464.5	1.6 / 9.4	1.0	0.0 / 7.0
+ 70		464.3	0.0 / 7.0		
+ 75	465.4	464.25	0.0 / 10.0	1.2	2.0 / 13.0
22	467.0	464.0	2.0 \ 13.0	3.0	4.0 / 16.0

It is not uncommon when the surface grade line is nearly at right angles to the axis of road-bed and the work not too heavy to assume that both fill and cut have a zero area at G, and the points A and B and sections $B\,F$ and $E\,D$ are ignored.

COMPUTING THE QUANTITIES.

The greatest work involved in computing the volumes lies in getting the areas of the cross sections.

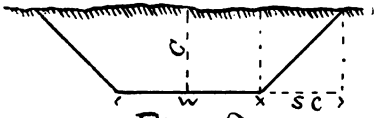


Fig. 9

The method of doing this from the form of notes already shown is as follows:

LEVEL SECTION.

Let s be the slope ratio $= \frac{hor.}{vert.}$, then

$$A = cw + sc^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Digitized by Google

THREE LEVEL SECTION.

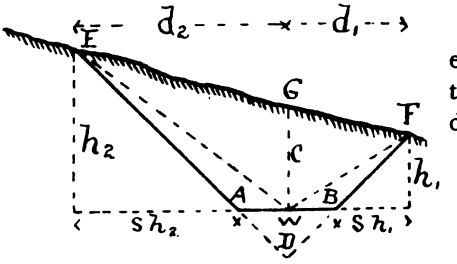


Fig. 10

The area is evidently equal to the sum of four triangles as shown by the dotted lines and is

$$A = \frac{c}{2}(d_1 + d_2) + \frac{w}{4}(h_1 + h_2) \ldots\ldots (7)$$

Since $$h_1 = \frac{d_1 - \frac{w}{2}}{s}$$

and $$h_2 = \frac{d_2 - \frac{w}{2}}{s}$$

we may, by substituting these values and placing $(d_1 + d_2) = D$, get

$$A = \frac{D}{2}\left(c + \frac{w}{2s}\right) - \frac{w^2}{4s} \ldots\ldots\ldots\ldots (8)$$

$\frac{w^2}{4s}$ is the area of the triangle ABD Fig. 10, and the first term is the area of the figure $EDFGE$.

Since $\frac{w}{2s}$ and $\frac{w^2}{4s}$ are constant quantities for any given length of cut or fill, equation (8) will give results with less work than equation (7).

IRREGULAR SECTION.

The area equals the sum of the areas of the trapezoids

$$ALKJ,\ JKME,\ EMNH,\ HNOB,$$

minus the triangles ALC and DOB.

This gives for the right side

$$A_1 = \left(\frac{c + h'}{2}\right) d' + \left(\frac{h' + h_1}{2}\right)(d_1 - d') - \left(d_1 - \frac{w}{2}\right)\frac{h_1}{2} \ldots\ldots (9)$$

Digitized by Google

and for the left side

$$A_2 = \left(\frac{c + h''}{2}\right) d'' + \left(\frac{h'' + h_2}{2}\right)(d_2 - d'') - \left(d_2 - \frac{w}{2}\right)\frac{h_2}{2} \quad \ldots\ldots (10)$$

The principle applies to any irregular section however many points there may be.

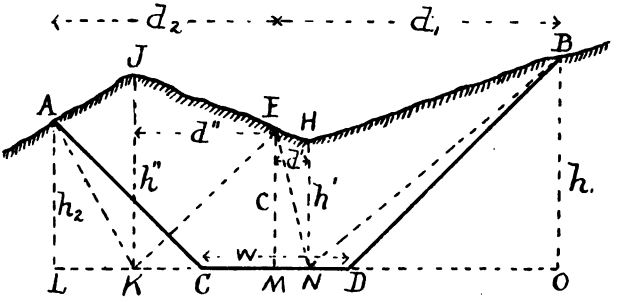


Fig. 11

In Fig. 11, the area may be divided into triangles thus:

Draw lines from E and B to N and from E and A to K; then

$$A = \tfrac{1}{2}\Big[c\,(d' + d'') + h'\,d_1 + h_1\,(d_1 - d') + h''\,d_2 + h_2\,(d_2 - d'') - h_1\left(d_1 - \frac{w}{2}\right) - h_2\left(d_2 - \frac{w}{2}\right)\Big] \quad \ldots\ldots (11)$$

These two forms are, of course, practically the same, and one may be obtained from the other.

Either of these expressions may be put in the following form by expanding and reducing:

$$A = \tfrac{1}{2}\Big[d'\,(c - h_1) + d_1\,(h' + h_1) + d''\,(c - d_2) + d_2\,(h'' + h_2) - h_1\left(d_1 - \frac{w}{2}\right) - h_2\left(d_2 - \frac{w}{2}\right)\Big] \quad \ldots\ldots (12)$$

The rule for the entire area including the triangles $D\,B\,O$ and $A\,C\,L$, as expressed by equation (12) is—taking each side separately and begining with the center—

Multiply the *distance* to each point taken, except the last, by the *difference* between the heights above grade of the *adjacent*

Digitized by Google

points (always subtracting the *following* from the *preceding*) and multiply the distance to the *slope stake* by the *sum* of the last two heights. Add the results and divide by two.

SIDE-HILL SECTION.

Reference to Fig. 6, will show that the area may be considered as two triangles, $A\,G\,C$ in fill, and $G\,D\,B$ in cut, or the right side may be considered as one side of a three level section and the remainder, two triangles, $G\,F\,E$ in cut, and $A\,C\,G$ in fill. It is unnecessary to write here the equations for these areas.

It is frequently convenient to know the amount of cut or fill in each one hundred feet or station and hence a cut or fill is usually computed one station at a time. When, in any particular case, there is no reason for this, considerable time and labor may be saved by computing the entire cut or fill at once.

Having computed the areas at the ends of each station, the volume of material in cubic yards in the station may be obtained by the method of average end areas from

$$V_{ea} = \frac{A_1 + A_2}{2} \cdot \frac{l}{27},$$

l being usually 100 feet,

$$V_{ea} = \frac{100}{54}(A_1 + A_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

By using the means of the dimensions of the end areas the middle area may be computed and the volume by the method of mean areas is

$$V_m = A_m \frac{100}{27} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

or by the prismoidal formula is

$$V_p = (A_1 + A_2 + 4\,A_m) \frac{100}{6.27} \ldots\ldots\ldots\ldots\ldots (15)$$

If it is desired to compute by either of Equations (14) or (15) and the sections are *irregular*, the middle section should be taken in the field; or the center height of *level sections* having the same areas as the end areas may be determined and the middle area assumed to be equal to a level area having a mean of these two heights for its center height. This might be called a *method of equivalent mean heights.*

Digitized by Google

In irregular earth-work, and in fact in any railway earth-work, it may be said that any one of the methods assumed gives only an approximate result and in any special case that method should be employed which, with the least expenditure of time, will secure a sufficiently accurate result.

As might be anticipated from Equations (4) and (5) the errors of the average end area method (the prismoidal formula being considered to give the most accurate result) and the mean area method vary in *level* sections as the *square of the differences of cut or fill*, and in *three level* sections as the *product of the differences of center cut or fill and the differences of total width.*

The method of mean areas entailing almost as much work as the prismoidal formula and less accurate in results is rarely used.

The method of average end areas being the simplest, and accurate—on light work, or heavy work having succeeding areas differing by small amounts—to probably within less than one per cent. is usually used. It is the legal method in some states.

On very heavy mountain work where slopes are steep the error of this method may probably reach or exceed two per cent. and in such work the prismoidal formula should be used.

By the graphical method, to be explained, the application of this formula is attended with very little labor.*

It has been stated that the average end area method should be used when end areas are not widely different and the prismoidal method when they are. It is not intended to imply that one station should be computed by one method and the next station by another, but that if the greater part of the work is of the class to which the average end area method will apply with sufficient exactness, then the whole should be computed by that method and vice versa.†

* There are other methods for approximating to the volume in special cases but those given are considered sufficient for practical use. For an interesting discussion of these methods and their relative accuracy reference may be made to Wellington's "Computation From Diagrams of Railway Earthwork."

† Let the student assume a number of stations with level cross sections differing in center height by one foot, two feet, &c., and compute the volumes by both methods and note the percentage of error. One set of such computations should be made for light sections and one for heavy sections, using for light sections center heights of from three to six feet and for heavy sections center heights from ten to thirty feet.

Digitized by Google

The cross sections may be drawn on cross section paper and the areas measured with the planimeter. With irregular areas this is considered by some the best method. When carried out as described later in this paper the writer prefers it to other methods.

When an embankment—usually called a "bank"—ends at the beginning of a trestle bridge the end of the bank is sloped off at the same slope made on the sides, and the "toe" thus formed should be computed by the prismoidal formula, the error in a ten foot bank being as much as 14% to 15% when computed by the average end area method, though the actual error in cubic yards is not great. The average end area method in this case gives results *too small.**

In the case of sidings added to the main track the cross section area would approximate that shown in Fig. 12.

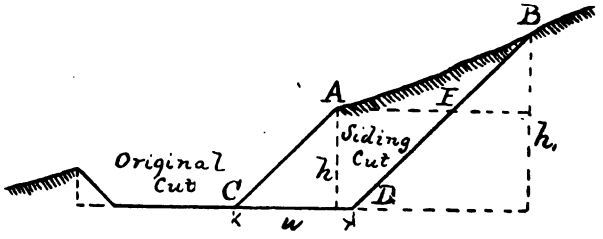


Fig. 12.

The area is evidently

$$A = \left(\frac{h + h_1}{2}\right) w \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Except near the ends of the siding where w is changing, the volumes in the siding addition would be correctly given by the average end area method since they are truncated wedges rather than truncated pyramids.

When an embankment is to be constructed and the adjacent cuts are too distant, or yield insufficient material, additional material is taken from shallow pits, called "borrow pits," excavated along the right of way outside the embankment. In such cases a space usually not less than six feet—called a "berm"—is left between the foot of the embankment and the edge of the borrow pit so that a section across the right of way might be like that shown in Fig. 13.

* Let the student draw a figure and show this.

Digitized by Google

The borrow pits are frequently taken out with vertical sides.

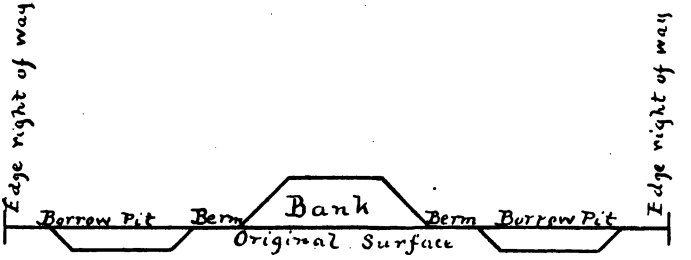


Fig. 13

Owing to changes in volume it is usual to measure all material in *excavation* and hence the borrow pits must be measured.

This is done by dividing them into squares or rectangles, taking elevations at each of the corners of these rectangles before and after excavation. The differences give the heights of the edges of rectangular truncated prisms whose volumes are computed as is shown in the discussion of Fig. 2, pp. 109.

In a series of such prisms adjacent to each other it is evident that in summing the expressions for the volumes of the separate prisms each corner height would be used as many times as there are prisms of which it is one edge. Therefore if in a number of such prisms the subscript numerals 1, 2, 3, &c., refer to the number of prisms of which each h is an edge the following will be the expression for the sum of all such adjacent prisms:

$$V = \frac{A}{4.27}(\Sigma h_1 + 2\,\Sigma h_2 + 3\,\Sigma h_3 + 4\,\Sigma h_4) \quad \ldots\ldots\ldots(17)$$

where A is the common right area of the prisms, Σ is the sign for summation, and the divisor 27 used to reduce to cubic yards.

If it will more nearly approximate the true volume to divide the surface into triangles, this may be done, and the subscripts may run as high as 8.*

The above methods are applicable to the computation of quantities in the graduation of large areas, as city lots or blocks.

The methods thus far given will suffice, if properly applied, for the computation of all earth-work executed in connection with the construction of roads, railroads or canals, so long as they are straight, or for general surface grading.

* Let the student make a diagram showing this.

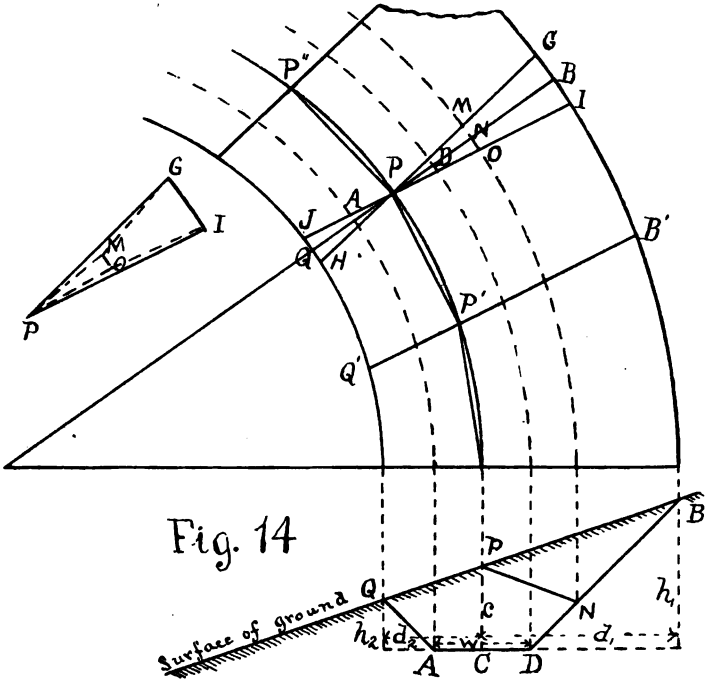
Digitized by Google

When a road is built on a curve, in order to more closely approximate the truth, a correction must be applied to the quantities obtained by the preceding methods.

This correction, called correction for curvature, arises from the fact that cross sections are taken perpendicular to the *tangent* to the curve at each station while the computations suppose them taken perpendicular to the *chord* between them.

The effect of this is shown in Fig. 14.

The cross section will be taken on the line $Q\,B$ (plan), while in computing it will be considered as taken on the line $J\,I$ for the station $P\,P'$, and on the line $H\,G$ for the station $P\,P''$. This results in omitting a doubly truncated prism $G\,I\,P$ (plan) of right section $P\,C\,D\,B$ (section) and in including twice a doubly truncated prism $J\,H\,P$ (plan) of right base $Q\,P\,C\,A$ (section).



In the figure the ground slopes towards the center of the curve, hence the omitted prism is greater than the twice included prism by a doubly truncated triangular prism of right

Digitized by Google

base $B\,N\,P$ (plan or section) and whose altitudes are—referring to the plan—$G\,I$ at B, $M\,O$ at N, and 0 at P. In the case shown, then, there should be added to each station a quantity equal to the volume of this prism.

This volume may be said to be

$$V_c = \text{area } B\,N\,P \times \frac{G\,I + M\,O + 0}{3} \quad \ldots\ldots\ldots (18)$$

$$\begin{aligned} \text{Area } P\,N\,B &= P\,C\,D\,B - P\,C\,D\,N \\ &= P\,C\,D\,B - P\,C\,A\,Q \\ &= \tfrac{1}{2}\,c\,(d_1 - d_2) + \tfrac{1}{4}\,w\,(h_1 - h_2). \end{aligned}$$

Let D be the degree of the curve.

Then $G\,I = 2d_1 \sin \tfrac{1}{2}D$. (Approx.)

And $M\,O = 2d_2 \sin \tfrac{1}{2}D$. (Approx.)

Substituting these values for $P\,N\,B$, $G\,I$, and $M\,O$, in Equation (18) and remembering that $\sin \tfrac{1}{2}D = \frac{50}{R}$, R being the radius of the curve, there results after a slight reduction—

$$V_c = \frac{100}{3\,R}\left(\tfrac{1}{2}\,c\,(d_1^2 - d_2^2) + \tfrac{1}{4}\,w\,(h_1 - h_2)\,(d_1 + d_2)\right)$$

substituting in which for $h_1 - h_2$ its value $\frac{d_1 - d_2}{s}$ there results

$$V_c = \frac{100}{3\,R}\,(d_1^2 - d_2^2)\left(\frac{c}{2} + \frac{w}{4\,s}\right) \quad \ldots\ldots\ldots\ldots\ldots (19)$$

This correction is additive when the highest ground is on the convex side of the curve and subtractive when the highest ground is on the concave side. This would be indicated by the sign of the correction.

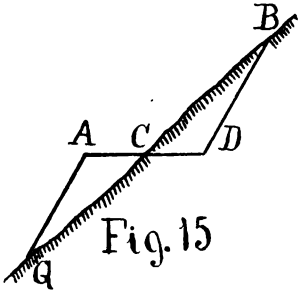


Fig. 15

The correction will be practically nothing in level sections and will increase as the slope of the ground across the road increases and will be relatively of most importance in side-hill work, sidings, and double track.

The three forms of side-hill sections that may arise are as shown in Figs. 15, 16 and 17.

Since it is usual to measure only excavation, the correction

Digitized by Google

prism for Fig. 15 will have a base CDB, for Fig. 16 BDN,

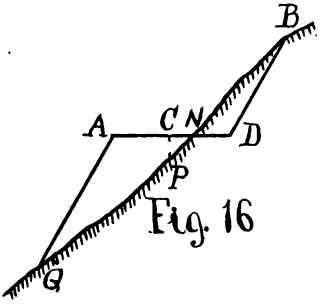


Fig. 16

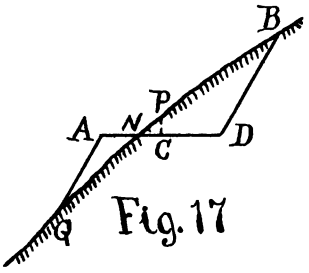


Fig. 17

and for Fig. 17 the base is BDN with altitude at N negative.

HAUL.

As has been stated, railway earth-work is usually paid for in excavation only, and good practice requires that the fills shall, so far as possible, be made from the adjacent cuts.

Should the adjacent cuts contain more material than is required in the fill, the latter may be widened with the waste material.

The surplus material is called "waste" and the irregular heaps in which it is sometimes piled alongside the cuts are called "spoil banks."

When the distance that the material in the cut must be hauled to its new position in the bank exceeds a certain limit (varying with the kind of material and topography) it becomes cheaper for the contractor to waste the remaining material in the cut, and build the remaining bank from borrow pits.

This practice is however not usually permitted by the specifications, which generally require that *all* the material in the cuts shall be placed in the adjacent banks. It would evidently be unfair to company and contractor alike, to pay a stipulated sum per cubic yard for all material of a given class excavated, regardless of the distance that it must be hauled, and so it is customary to specify that the price bid for excavation shall include payment for placing the material in the adjacent banks wherever the distance hauled is less than a certain specified limit—varying from 200 feet to 500 feet or even 1,000 feet—and that for all material hauled beyond this distance, a certain specified sum per cubic yard per station (100 ft.) hauled beyond

Digitized by Google

the specified limit of free haul, shall be paid *in addition* to the regular price per cubic yard for excavation.

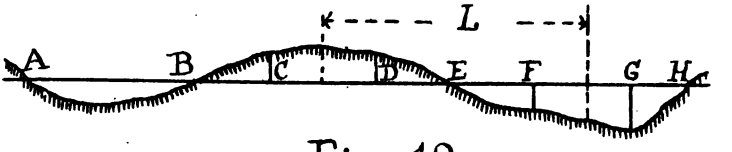


Fig. 18

Let Fig. 18 be a profile of the center line of a railroad; the long straight line is the "grade line" and the irregular line the surface.

Assume that the portion of cut DE makes the fill EF, and that the distance DF is the limit of free haul. Assume also that the portion of cut BC is placed in the bank at the left, and that the remaining portion of the cut CD makes the bank FG.

The positions of the transverse planes D and F are determined by trial.

Then if V_1 be any small portion of the volume CD, (say 1 cubic yard) and l_1 the distance from the position of its center of gravity in CD to its new position in FG, then $\frac{l_1 - DF}{100}$ = distance in stations V_1 is "overhauled." Similarly $\frac{l_2 - DF}{100}$ = distance in stations V_2 is overhauled; and if there be n cubic yards in CD, then the total distance in stations that *one* cubic yard is overhauled is

$$O = \frac{l_1 - DF + l_2 - DF + l_3 - DF + \&c., \; l_n - DF}{100}$$

$$= \frac{l_1 + l_2 + \&c, \; l_n - n\,DF}{100} = \frac{\Sigma l - n\,DF}{100}$$

and this quantity, multiplied by the specified price per cubic yard per station overhauled, will give the extra sum due the contractor for moving CD to FG. But it is practically impossible to determine the distance each yard is moved, and it is unnecessary; for the same result may be obtained by multiplying the *average* distance that the whole quantity CD is overhauled, by its volume in cubic yards.

This average distance is the distance from the transverse plane containing the center of gravity of CD to the transverse

Digitized by Google

plane containing the center of gravity of FG, *less* the distance DF.

The only difficulty in determining the distance L, Fig 18, is the difficulty of determining the positions of the transverse planes containing the centers of gravity of the portions CD and FG. This may be easily done when it is remembered that the distance from one end of any prismoid to the transverse section containing its center of gravity is expressed by the following formula:

$$X = \frac{l}{2} + \frac{l^2}{12}\frac{(A_2 - A_1)}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which l is the length of prismoid, A_2 the end area farthest from the end considered as the origin, A_1 the area of the end considered as the origin, and V the volume of the prismoid. All quantities are in feet. If the volume is given in cubic yards the expression (Eq. 20) is again divided by 27.

Equation (20) may be derived as follows—

Assume in Fig. 19 a prismoid, which for convenience of conception, may be assumed to have similar figures for its two end areas.

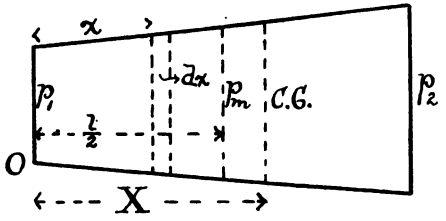


Fig. 19

Then since the areas are as the squares of their homologous sides there may be written:

$$A_1 = Kp_1^2$$

$$A_2 = Kp_2^2$$

$$A_m = Kp_m^2 = K\left(\frac{p_1 + p_2}{2}\right)^2$$

and if K is assumed equal to 1, which simply means that the solid is a square sectioned truncated pyramid, there may be written:

$$A_1 = p_1^2,\ A_2 = p_2^2,\ A_m = \left(\frac{p_1 + p_2}{2}\right)^2$$

$$A_x = \left(p_1 + \frac{x}{l}(p_2 - p_1)\right)^2.$$

The entire volume is

$$V = \frac{l}{6}\left(p_1^2 + 4\left(\frac{p_1 + p_2}{2}\right)^2 + p_2^2\right)$$

$$= \frac{l}{6}\left(p_1^2 + (p_1 + p_2)^2 + p_2^2\right) \quad \ldots\ldots\ldots\ldots (21)$$

Digitized by Google

The moment of the entire volume about O is

$$M = X\frac{l}{6}\left(p_1^2 + (p_1+p_2)^2 + p_2^2\right) \quad \ldots\ldots\ldots\ldots(22)$$

The moment of the differential volume $A_x\,dx$ is

$$dM = x\,A_x\,dx = x\,dx\left(p_1 + \frac{x}{l}(p_2 - p_1)\right)^2$$

which integrated between the limits 0 and l gives—

$$\begin{aligned} M &= \int_o^l x\left(p_1 + \frac{x}{l}(p_2 - p_1)\right)^2 dx \\ &= \frac{l^2}{12}(p_1^2 + 2p_1p_2 + 3p_2^2) \\ &= \frac{l^2}{12}\left((p_1^2 + 2p_1p_2 + p_2^2) + 2p_2^2\right) \\ &= \frac{l^2}{12}\left((p_1+p_2)^2 + 2p_2^2\right). \end{aligned}$$

But, from Eq. (22),

$$\begin{aligned} M &= \frac{2}{12}l\left(p_1^2 + (p_1+p_2)^2 + p_2^2\right)X \\ &= \frac{l^2}{12}\left((p_1+p_2)^2 + 2p_2^2\right) \end{aligned}$$

whence,

$$X = \frac{l}{2}\cdot\frac{(p_1+p_2)^2 + 2p_2^2}{p_1^2 + (p_1+p_2)^2 + p_2^2} \quad \ldots\ldots\ldots\ldots\ldots(23)$$

Now X is greater than $\frac{l}{2}$ by

$$\begin{aligned} C = X - \frac{l}{2} &= \frac{l}{2}\left(\frac{(p_1+p_2)^2 + 2p_2^2}{p_1^2 + (p_1+p_2)^2 + p_2^2} - 1\right) \\ &= \frac{l}{2}\left(\frac{p_2^2 - p_1^2}{p_1^2 + (p_1+p_2)^2 + p_2^2}\right) \quad \ldots\ldots(24) \end{aligned}$$

In Eq. (24) the numerator of the fraction in parenthesis is $A_2 - A_1$ and the denominator is $V\frac{6}{l}$.

Substituting these values,

$$C = \frac{l^2}{12}\left(\frac{A_2 - A_1}{V}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(25)$$

Digitized by Google

Therefore,

$$X = \frac{l}{2} + \frac{l^2}{12}\left(\frac{A_2 - A_1}{V}\right) \quad \ldots\ldots\ldots(26)\ Q.\ E.\ D.$$

When V is given in cubic yards, equation (26) becomes—

$$X = \frac{l}{2} + \frac{l^2}{12}\left(\frac{A_2 - A_1}{27\,V}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(27)$$

This same result might have been obtained for a more general form of prismoid, by using the form for areas and volumes given in Gillespie's "Roads and Railroads," instead of assuming the simple square sectioned truncated pyramid. If the section of cut $C\,D$ Fig. 18, is a series of prismoids, *i. e.*, several stations long, the center of gravity of each may be found by Eq. (27), and the resulting center of gravity of the series determined by the well-known principle of moments. But this may be more easily done when the prismoids are of equal length. In this case, first assume the center of gravity of each to be in its mid-section, giving the following expression for the distance of center of gravity from one end, assuming a special case for definiteness.

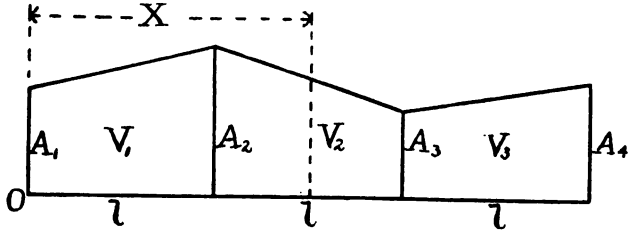


Fig. 20

Referring to Fig. 20,

$$X_{\frac{l}{2}} = \frac{\frac{l}{2}(V_1 + 3V_2 + 5V_3)}{\Sigma V}.$$

If C_1, C_2, C_3, &c., represent corrections as found in Eq. (25), then

$$X = \frac{V_1\left(\frac{l}{2} + C_1\right) + V_2\left(\frac{3}{2}l + C_2\right) + V_3\left(\frac{5}{2}l + C_3\right)}{\Sigma V}$$

$$X - X_{\frac{l}{2}} = \frac{V_1 C_1 + V_2 C_2 + V_3 C_3}{\Sigma V}.$$

Digitized by Google

Substituting values of C_1, C_2, and C_3 from Eq. (25),

$$X - X_{\frac{l}{2}} = \frac{\frac{l^2}{12}\left(\frac{A_2 - A_1}{V_1} V_1 + \frac{A_3 - A_2}{V_2} V_2 + \frac{A_4 - A_3}{V_3} V_3\right)}{\Sigma V}$$

$$= \frac{l^2}{12} \frac{A_4 - A_1}{\Sigma V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

which indicates that the same correction holds good for a series of volumes as for a single volume, provided the lengths are equal. This gives us the following rule for finding the center of gravity of a prismoid or a series of prismoids of equal length:

Determine distance of the center of gravity from one end, assuming that the center of gravity of each prismoid is in the mid-section. Add to the result thus found the product, one-twelfth the square of the length of a single prismoid into the difference of the extreme end areas (always subtracting the first from the last) divided by the total volume of all the prismoids.

If the volumes are given in cubic yards always divide again by 27.

Since the areas of the cross sections at the ends of each station and the volume of each station are always known, the center of gravity of any series of stations as CD or FG Fig. 18 may be thus easily determined and the overhaul computed as previously explained. Equation (27) is an extremely useful one, for it applies to all solids that may be computed by the prismoidal formula, and this includes all the regular solids of revolution.*

ESTIMATES.

In railway work it is usual to make three computations of the quantities of the various materials removed or to be removed.

These are: 1. The preliminary estimate, based on information furnished by the preliminary survey. 2. The location estimate, based on information furnished by the location. 3. The final estimate, based on the work as actually done.

The methods already given for computing volumes from cross section notes are used in making the final estimate. Instead, however, of performing all the numerical operations indicated

*Let the student show its applicability to a cone of altitude a, and radius of base b, and a hemisphere of radius r.

Digitized by Google

in what has gone before, the work may be much lessened by the use of tables or diagrams, and in the case of preliminary and location estimates the tables or diagrams should always be used.

According to the method of making the preliminary survey, one or another method may be used for making the estimate of materials.

If the preliminary survey is merely a line, run as near to its proper place as the chief of the party may be able to judge, along which elevations have been taken at each station or oftener, as the form of the ground may render necessary, the procedure is something as follows :—

From the leveler's notes, a profile of the line is made on profile paper, and a grade line also drawn. This grade line will be put on by the chief of party.

The vertical distance at any station, between the surface line and grade line is the depth of center cut or fill at that station. If the ground is fairly level transversely of the line, the sections may be assumed to be level sections, and the volume for a single station may be computed from the formula—

$$Q = \frac{l}{27}(wc + sc^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

in which w is width of road-bed, c is center height, and s is slope ratio.

This equation may be tabulated or diagramed as hereafter to be shown. The value of c is taken midway between two station points. If the ground is not nearly level across the line the transverse angle of inclination should be taken by the leveler at each station point, and the quantity may be computed by the following formula:

$$Q = \frac{l}{27}\left(\frac{C^2 \cdot 2sr^2}{r^2 - s^2}\right) - \frac{l}{27} \cdot \frac{w^2}{4s} \quad \ldots\ldots\ldots\ldots (30)$$

in which C is the center height plus the height of the grade prism, s is the slope ratio of the sides, r is the ratio $\left(\frac{hor}{vert}\right)$ of the surface slope, and w is the width of road-bed.

The foregoing equations assume the average end area method to be used, and as this method tends to excess and is simpler of

Digitized by Google

application than the prismoidal formula, its use on preliminary and location estimates is advisable, even though it is deemed necessary to use the prismoidal formula for the final estimates.

Location estimates may be made by either equation (29) or equation (30), as indicated by the character of the ground.

When the preliminary survey partakes of the nature of a preliminary location, and from it contour maps are prepared, on which the preliminary location is drawn, a new profile of the projected location may be drawn from the contour map and a grade line placed on it. Center heights of cut and fill may now be taken from this profile, and transverse slopes from the contour map, and a very close preliminary estimate thus prepared.

When the final location is made on the ground, the profile is drawn from the actual level notes, and the transverse slopes determined in the field. The method of determining these slopes is this: the rodman lays his rod on the ground across the line, and places on it a pocket clinometer, brings the bubble to the center of the tube, and then reads the angle of slope, and calls it out to the leveler who records it. The slope ratio used in these notes is the cotangent of the slope angle.

The derivation of equation (30) is as follows—Fig. 21—

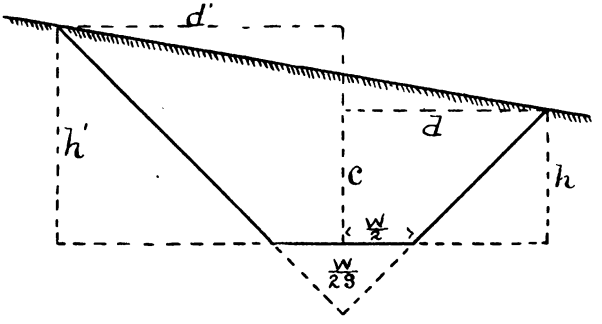


Fig. 21

The ordinary equation for the area of a three level section is

$$A = \left(c + \frac{w}{2s}\right) D - \frac{w^2}{4s} \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

$$D = d' + d.$$

Digitized by Google

If s be slope ratio of the sides and r the slope ratio of the surface, then—

$$c + \frac{d'}{r} = h' = \frac{d' - \frac{w}{2}}{s},$$

whence

$$d' = \frac{\left(c + \frac{w}{2s}\right) rs}{r - s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Similarly,

$$d = \frac{\left(c + \frac{w}{2s}\right) rs}{r + s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

and here the work may stop if it is desired to handle these comparatively simple numbers, substituting the values of d' and d for D in Eq. (31). To get the final equation (30), substitute as above and there results—

$$A = \left(c + \frac{w}{2s}\right)\left(c + \frac{w}{2s}\right)\left(\frac{1}{r - s} + \frac{1}{r + s}\right) rs - \frac{w^2}{4s}.$$

Substituting for $\left(c + \frac{w}{2s}\right)$ a value C, there results—

$$A = \frac{C^2 2r^2 s}{r^2 - s^2} - \frac{w^2}{4s}$$

and

$$Q = \frac{l}{27}\left(\frac{C^2 2r^2 s}{r^2 - s^2}\right) - \frac{l}{27} \cdot \frac{w^2}{4s}.$$

If the slopes on the left and right of the center are not the same, the d's would be found as in equations (32) and (33), the method being precisely the same except that there would be two values for r, one for each side. If anything like a close estimate is desired in fairly uneven country, the transverse slope of the ground should be considered.

The work of computing quantities by the formulas that have thus far been given may be greatly lessened by making tables of volumes for various depth of cut or fill, a separate table being required for each width of road-bed and side slope.

Digitized by Google

The work may be still further lessened by resorting to diagrams which are merely graphical representations of the equations themselves.

TABLES.

Tables may be computed for either average end area or prismoidal methods.

The average end area tables for level cross sections are all practically alike and Rice's Tables, Trautwine's Tables, or others, may be used.

The tables devised by different authors for the prismoidal formula are all different. Probably the best are those of Prof. Johnson or Prof. Crandall.

The method of making the average end area tables for *level* sections is as follows:—

The equation for the volume of a one hundred foot prismoid, having width of base w, center height c and slope ratio s is—

$$Q = \frac{100}{27}(w\,c + s\,c^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

For given values of w and s, Q may be computed for values of c varying by 0.1 ft., and entered in a table of the following form:

TABLE OF QUANTITIES IN CUBIC YARDS.

Prismoids 100 feet long; base 20 feet; slope 1 hor. to 1 vert.

Depth of Cut Feet.	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	cu. yds.	cu. yds.	cu. yds.	cu. yds.	cu. yds.	cu. yds.	cu. yds.	cu. yds.	cu. yds.	cu. yds.
0										
1										
2										
3										
&c.										

The labor of computing such a table may be greatly reduced by noting that Eq. (34) is an equation of the second degree, and that, therefore, its second differential is a constant quantity.

Digitized by Google

Differentiating equation (34):—

$$dQ = \frac{100}{27}(w + 2sc)\,dc$$

$$d^2Q = \frac{100}{27} \cdot 2s\,dc^2, \quad \text{a constant.}$$

Therefore, compute the volume for $c = 0.1$, and for $c = 0.2$: to the difference between these two add $\frac{100}{27} \cdot 2s\,dc^2$. Add the sum to the volume for $c = 0.2$ and there results the volume for $c = 0.3$.

To the difference between values for $c = 0.2$ and $c = 0.3$, add again $\frac{100}{27} 2s\,dc^2$, and the resulting sum added to volume for $c = 0.3$ gives the volume for $c = 0.4$, and so on. To do this conveniently prepare three columns as shown:

c.	Volume.	First Difference.
0.1	7.44	
		7.519
0.2	14.96	.074
		7.593
0.3	22.55	.074
		7.667
0.4	30.22	.074
		7.741
0.5	37.96	
&c.	&c.	&c.

Assume, for example, values as in the heading to the form for table just given, viz: $w = 20 \quad s = 1$.

Then for $c = 0.1$, $Q = \frac{100}{27}(wc + sc^2) =$	7.444 +
for $c = 0.2$, $Q =$	14 963 —
First difference,	7.519
The second difference is $d^2Q = \frac{100}{27} \cdot 2s\,dc^2 =$	.074074
New first difference	= 7 593

It was unnecessary to compute for $c = 0.2$, as the value of

Digitized by Google

$d^2\ Q$ might have been added at once to 7.444 to get the first difference, 7.519.

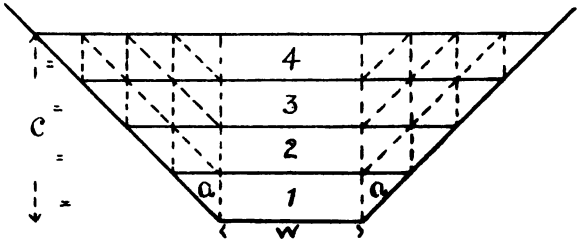


Fig. 22

A clear geometric conception of these quantities may be obtained from Fig. 22, from which it is evident that for

	1st diff.	2d diff.
$c = 1$, area $= w + 2a$		
	$w + 6a$	
$c = 2$, area $= 2w + 8a$		$4a$
	$w + 10a$	
$c = 3$, area $= 3w + 18a$		$4a$
	$w + 14a$	
$c = 4$, area $= 4w + 32a$		&c.

Tables for the prismoidal formula, while very simple in manner of construction, must cover a large amount of space if the amount of numerical computation incident to their use is to be reduced to a minimum, and it is thought that the diagram hereafter to be described involves less labor in computing, and requires less space for the same amount of labor than the tables. The results obtained by its use are as exact as the field methods warrant.

PLANIMETER.

The volume of a station, or series of stations, of cut or fill, may be found by drawing the areas on cross section paper, measuring the areas with the planimeter, and then applying the length and computing by either the prismoidal or average end area method.

To do this proceed as follows:—

Assume a scale, say ten feet per inch, that of the paper. Lay off a line equal to width of road-bed; draw from each end of

Digitized by Google

this line a line, making with it the slope angle, to represent the sides of the cut or fill. Use this base for as many successive areas as can be plotted without confusion. Only the surface lines will have to be drawn from the field notes.

Now for average end area method, place the planimeter in a convenient position with the tracing point at one extremity of the base line and read the wheel. Circumscribe the first area, and, without stopping, circumscribe the second and read the wheel. The difference in the two readings will be the record due to the sum of the two areas.

If l = the length of the graduated arm of the planimeter from pivot to tracing point, c the circumference of the wheel in inches, and n the number of revolutions made by the wheel in circumscribing a given area, *i. e.*, the record of the wheel, then the area is given by

$$A = l \times n \times c.$$

If c is not known it may be found by circumscribing a known area with a known length of arm and noting the record n of the wheel, when

$$c = \frac{A}{l \times n}.$$

Now in the above measurement of two areas we have the sum of the two areas

$$A_1 + A_2 = lnc$$

$$\frac{A_1 + A_2}{2} = \frac{lnc}{2}.$$

This is the result in *square inches*. But each square inch is 100 square feet, hence in square feet there follows:

$$\frac{A_1 + A_2}{2} \text{ sq. ft.} = \frac{100\, lnc}{2}.$$

This, again, multiplied by 100, the length of the station in feet, and divided by twenty-seven, to reduce to cubic yards, gives for the volume—

$$V = \frac{100 \times 100\, lnc}{2 \times 27}$$

$$= (185.185 +)\, lnc.$$

Digitized by Google

But it is desired to get the volume from the reading of the planimeter without excessive work, and therefore a suitable length of arm, l, should be used, such that a simple multiplication of the reading, n, will give the volume. Let it be assumed that a single revolution of the wheel shall correspond to 2,000 cubic yards. Then—

$$2000 = 185.185.\, lc.$$

Since c is usually between two and three inches, this will give a convenient value for l. In the instrument used by the writer, the value of c is 2.31 inches.

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

Substituting this value of c there results—

$$l = 4.675 \text{ inches.}$$

In this instrument, therefore, the volume is given at once by—

$$V = 2{,}000n$$

when $l = 4.675$ inches.

Since the revolutions are read to the one-thousandth of a revolution there results the rule for this instrument:—

The record of the wheel, without decimal point, multiplied by 2, gives the volume in cubic yards.

If the arm of the planimeter cannot be set, as is the case with cheaper grades, then only square inches may be read, and the arm and wheel are usually so made that a single revolution of the wheel corresponds to ten square inches. Thus—

$$A \text{ sq. in.} = 10n.$$

In this case measure each area separately, and multiply it by $\frac{50}{27}$, and apply the result to each of the stations that lie on either side of the area. The volume may be taken from a diagram consisting of a single straight line with axis of abscissas as axis of A and tangent of angle made with this axis $= \frac{50}{27}$.

If the planimeter is used, and it is desirable to compute by the prismoidal formula, the two end areas are drawn from the same base, and the middle area may then be drawn by interpolation, or the tracing point may ordinarily, with sufficient exactness, be passed by eye midway between the two lines forming the surface lines of the end areas.

The tracer should be passed around the first area once, the

Digitized by Google

middle area four times, and the second area once, and the resulting record, divided by 6 and 27, and multiplied by 100, gives the volume for a station.

The arm may be determined for this case, as well as with average end areas, to give a minimum of labor.

To measure, by the average end area method, a series of volumes of equal length at once, draw the end areas on one base, and placing the planimeter and reading the wheel, pass the tracing point around the first and last areas once, and around all the others twice, and then read the wheel, when the difference in readings, multiplied by the common length and divided by 27, gives the volume in cubic yards.

In case the arm has been set as described in the example given for average end area method, the record of the wheel without decimal point, multiplied by 2, is the volume of the series in cubic yards. The same reasoning may be applied to the prismoidal method.

The method with the planimeter has its maximum efficiency in the case of irregular sections, but may be used to advantage with three level sections if no diagram is at hand. The method by diagrams is considered by the writer the best of all methods for this work. The student should, at all times, look for short methods for performing such calculations as he has to make, and should accept any method lessening routine labor, that gives sufficient exactness for the work in hand.

DIAGRAMS.*

Any equation between two variables may be drawn on cross section paper, so that all probable values of one variable for given values of the other variable may be scaled from the diagram thus prepared. This principle is the basis of the most rapid and easy method known to the writer for computing volumes in railway earth-work.

The only difficulty in constructing the various equations is in separating an equation that is apparently one between three or more variables, into several equations each between two variables, and this is merely a matter of ingenious arrangement. Mr. A. M. Wellington is the only author who has published

*Students should make to scale one of each of the diagrams described. The scale need not be large.

Digitized by Google

diagrams, made in this way, on a working scale.* Trautwine has also published diagrams made in a different way. The late Prof. Chas. A. Smith read a paper before the Engineers' Club of St. Louis, describing the method of making such diagrams, and frequently made and used such diagrams in his professional work. The writer has used Wellington's diagrams, and others, for other purposes, constructed by himself, with great saving of labor, and believes that they should always be used when it is possible to read results to a degree of accuracy equal to that obtaining in the methods employed in doing the work to be measured.

The use of diagrams for all sorts of calculations, except earth-work, is quite common, and there seems to be no reason why the earth-work diagrams should be slighted.

Assume an equation of the first degree—

$$y = ax.$$

This is the equation of a straight line passing through the origin of co-ordinates, and y may be supposed to be simply the product of two numbers, one of which is a variable; or it may represent the areas of various rectangles having a constant side; or it may represent the area of a series of triangles having a constant base or altitude.

The width of the strip of land taken for right of way, by railroads, is usually one hundred feet. Each owner, through whose land the line passes, is paid for his land by the acre.

If the strip is 100 feet wide the acreage per station will be $\frac{100 \times 100}{43560} = 0.2295\,A.$

If there are n stations of the line on the land of a single individual, the acreage taken from him is $A = .2295\,n.$

A diagram may be made to rapidly compute the value of A for varying values of n. On a piece of cross section paper lay off from one corner along one side, a distance to scale, say one inch per station, equal to the greatest number of stations likely to affect one individual—say 20 inches. On the perpendicular, at this point, lay off to convenient scale, say one inch per half acre, a distance equal to $.2295\,n$, in the example assumed 9.18 inches. From this point draw a line to the point of origin. An

*"*Computation from Diagrams of Railway Earth-work.*"

Digitized by Google

ordinate to this line at any point on the axis of n gives, when read by the scale used in laying off the acres on the extreme ordinate, the acreage due to the number of stations n, at the point at which the ordinate is taken.

An equation of the form—

$$y = ax\,z,$$

while an equation between three variables, may be conceived for diagraming to be made up of n equations of the form—

$$y = ab\,z,$$

b having a different value in each equation. A diagram for such an equation is made as follows—Fig. 23—

Assume a value, n, for z as large as is likely to be needed, and lay it off to scale on axis of z. Assume now a small value for x, say 0.1 or 1.0, or more, according to the purpose of the diagram, and compute a value for y and lay it off to scale, say to y'. Draw the line Oy'. Ordinates to this line will give values of y for the one assumed value of x and varying values of z.

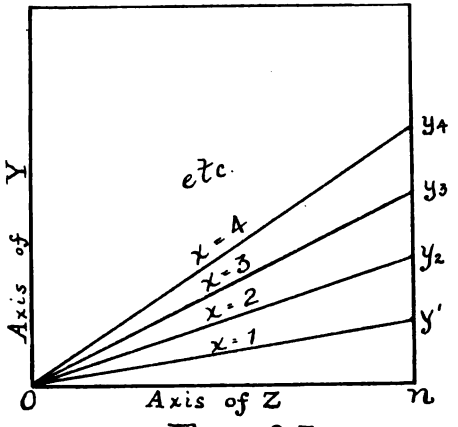


Fig. 23

Assume another value for x and draw another line, &c.; mark these lines with the value of x assumed for each. The values of x should be taken sufficiently close together, so that values of y, for intermediate values of x, may be taken by interpolation with sufficient accuracy for the purpose in hand.

Digitized by Google

Example—Let it be desired to make a diagram to determine approximately the area of a series of rectangles with varying sides.

The equation is $y_1 = xz$.

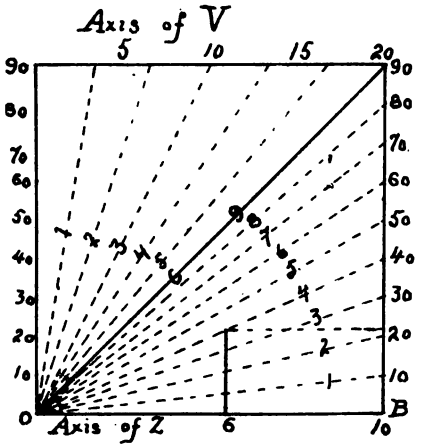


Fig. 24.

In Fig. 24 lay off to scale on the axis of z, a value as large as is likely to be needed for z, say 10 units (assume scale of 1 inch per foot), to B. For $x = 1$, $y = 10$. Lay off 10 units to scale from B towards C. (Assume scale 1 inch for 10 square feet.) Proceed similarly for $x = 2$, 3, 4, &c., to 9, assumed the largest probable value of x for the purpose of the diagram. Draw lines from the points thus found on BC to A, and, if desired, draw lines between these for interpolation. To determine area of rectangle 6 × 4 feet. At point 6 on AB follow up the perpendicular to the line of $z = 4$ and carry horizontally over to the line BC or AD and read the value of y. The greater value of the diagram is for values of x and z expressed in mixed numbers. Assume now the equation—

$$V_{cy} = \frac{u\,y}{27},$$

which is, of course, equivalent to—

$$V_{cy} = \frac{u\,x\,z}{27},$$

and gives the volume, in cubic yards, of a cubical block of

Digitized by Google

dimensions u, x, and z expressed in feet. In Fig. 24 lay off on $A\,D$ values of y likely to be needed for the purpose of the diagram, and on the perpendicular at the point of greatest probable value—say 90—lay off values to scale of V, for $u = 1$, $u = 2$, $u = 3$, etc., to the largest probable value for u, say 6. Draw lines from the points thus found to A. Ordinates from the axis of y to the line representing a given value of u, give, when read to the scale used along the axis of V, the corresponding volume in cubic yards.

The diagram $A\,C\,D$ is an entirely distinct diagram from $A\,C\,B$, and need not be combined with it. As it is convenient to so combine it, when possible, the maximum quantities selected for the example have been (and must always in such case be) chosen to give a convenient volume in cubic yards.

$$V_{cy} = \frac{10 \times 9 \times 6}{27} = 20 \text{ cubic yards.}$$

These quantities were used in the first diagram of this kind of which the writer has any knowledge, and which was used to rapidly approximate the quantity of stone delivered monthly for some extensive bridge work.

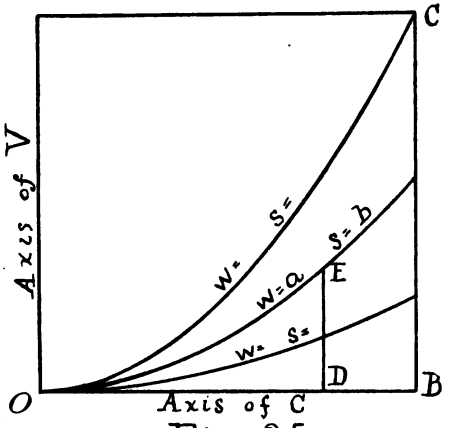


Fig. 25

The methods for diagraming the various equations already deduced for railway earth-work will now be shown.

First—level cross section—average end area method

$$V_{cy} = \frac{l}{27}(cw + sc^2).$$

Digitized by Google

The line representing this equation for given values of w and s may be drawn as in Fig. 25, as many lines for each value of w likely to be used, as there are values for s. If there were 3 values for w, and 2 values for s for each value of w, there would be six lines.

If $c = 0$, $V = 0$, and the lines all pass through the origin. Compute one other value for V for each of the lines to be drawn, and with the largest probable value of c. Plot this value of V. Then any of the ordinary methods of drawing a parabola may be used to draw the line.

Then the volume for a given value of $c = OD$, and of $w = a$, and $s = b$, is given to scale by the ordinate DE.

This same equation may be diagramed differently.

$$V_{cy} = \frac{l}{27}\left(cw + sc^2\right) = \frac{l}{27}cw + \frac{l}{27}sc^2.$$

The first term of the second member represents a straight line and the second term a parabola.

In Fig. 26 draw as before for each probable value of s, a parabola, and for each probable value of w, a straight line, and since the ordinates to these lines, for given values of c, are to be added, draw one *above* the axis of c and the other *below* that axis.

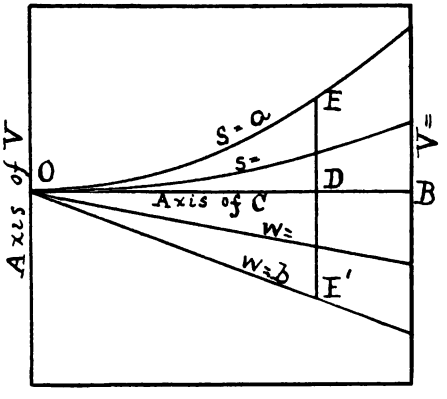


Fig. 26

For any value, OD of c, and a of s, and b of w, the volume is, to scale used for volume in drawing the lines, the ordinate EE', which may be taken on dividers or a slip of paper

Digitized by Google

and applied to the scale of volumes so as to read the whole from a zero point. This is not so convenient a diagram and is given merely to show the method of dividing an equation.

For three-level section, equation (8)—

$$A = \frac{D}{2}\left(c + \frac{w}{2s}\right) - \frac{w^2}{4s}$$

and

$$V_{cy} = \frac{l}{27} \times \frac{D}{2}\left(c + \frac{w}{2s}\right) - \frac{l}{27} \times \frac{w^2}{4s}.$$

The first term of the second member is, for a given value of c, a straight line, and for varying values of c may be represented by a series of straight lines, one for each value of c.

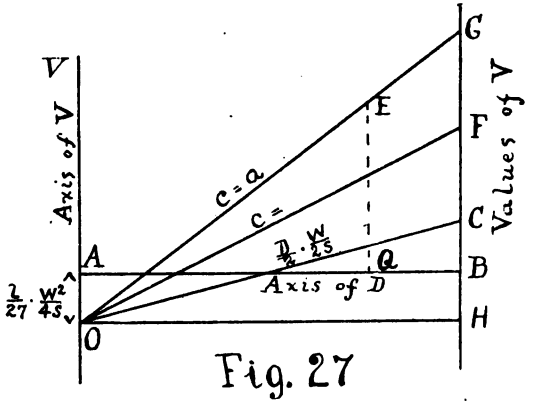


Fig. 27

The second term is, for given values of w and s, a constant quantity, and it will be considered that for each different value of these quantities a diagram is to be constructed. The first term may further be divided into—

$$\frac{l}{27} \times \frac{D}{2}c + \frac{l}{27} \times \frac{D}{2} \times \frac{w}{2s}$$

and each of these terms diagramed separately, the second being represented by a single straight line, and the first by a series of straight lines, one for each value of c.

To draw the diagram, Fig. 27, assume axis of $D = AB$, and of $V = AV$. To scale to be used for V, lay off below AB a distance $= \frac{l}{27} \times \frac{w^2}{4s} = AO$, and draw a line parallel to AB.

Digitized by Google

Ordinates to this line for any value of D give the value of the subtractive term.

Now assume O as the origin, and for the largest probable value of $D = AB$, compute $\frac{l}{27} \times \frac{w}{2s} \times \frac{D}{2}$, and lay it off from H to C. Draw the line CO. Ordinates to this line from the line OH represent values of $\frac{l}{27} \times \frac{w}{2s} \times \frac{D}{2}$, when taken to the scale used for V, and ordinates to this line from AB represent the difference,

$$\frac{l}{27} \times \frac{D}{2} \times \frac{w}{2s} - \frac{l}{27} \times \frac{w^2}{4s}$$

to which is yet to be added

$$\frac{l}{27} \times \frac{D}{2} c.$$

Compute this quantity for the largest probable value of c, corresponding to the value AB of D, and lay the result off to scale of V from C, say to G. Draw the line GO. Ordinates to this line from the line OC give to scale of V for various values of D and the assumed value of c, the quantity

$$\frac{l}{27} \times \frac{D}{2} c$$

and hence for the assumed value of c, and any value of D, as AQ, the volume V is given by the ordinate QE. Since for a given value of D the term $\frac{l}{27} \times \frac{D}{2} c$ varies directly with the values of c, the distance CG may be divided into n equal parts, and lines drawn from each point of division to O, when the ordinates to these lines will represent the value of $\frac{l}{27} \times \frac{D}{2} c$ for values of c (counting upward), $\frac{a}{n}$, $\frac{2a}{n}$, $\frac{3a}{n}$, &c., and may be so lettered. The interval between two lines will be determined by the scale of the diagram and that again by the accuracy desired in the final result.

The foregoing equation—

$$\frac{l}{27}\left[\frac{D}{2}\left(c + \frac{w}{2s}\right) - \frac{w^2}{4s}\right]$$

Digitized by Google

is used for volumes by the average end area method as follows—

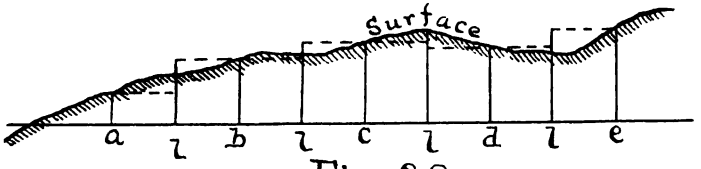


Fig. 28

Let a, b, c, &c., Fig. 28, represent areas of cross sections at the ends of the various stations. Then—

$$V = \frac{l}{27}\left(\frac{a+b}{2}\right) + \frac{l}{27}\left(\frac{b+c}{2}\right) + \frac{l}{27}\left(\frac{c+d}{2}\right) + \frac{l}{27}\left(\frac{d+e}{2}\right) \text{ &c.}$$

$$= \frac{l}{27}\frac{(a+e)}{2} + \frac{l}{27}(b+c+d, \text{ &c.})$$

Therefore, for the cross section at a, and for the half station on its right, take the volume from the diagram and divide it by 2. For the remaining stations, take the quantity directly from the diagram. There is thus obtained the volume for fifty feet (or a half station length) on either side of a station stake. As it is usual to compute the volume from between adjacent station points, it is customary to take but half the solidity due to any area, computed for a full station, and add this quantity to that due to a half station of the next area. Thus the diagram is made by Mr. Wellington to give volumes for a half station only, and each result must be used twice—once on each side of the station point of which the area is considered. For fractional stations, the quantity is found by taking from the diagram the quantity for the given cross section, for a half station, and then taking the proportion of that quantity that the fraction of a station is of a half station. This method has another advantage, viz: For the same scale it makes the precision of the diagram greater, or with a given minimum unit for reading and a given scale it decreases the size of sheet. Since the diagram, when constructed for use, would be too large for convenience, it is divided into sections as shown in Figs. 29 and 30.

It is evident that there would be no probability that a value of $c = 1$ would occur with a value of $D = 50$; for instance, since corresponding values of D and c in level sections and

Digitized by Google

3 level sections of approximately the same area, are not usually widely different, the diagram is cut as shown in Fig. 29, so that each section AB, CD, &c., shall include those values of c

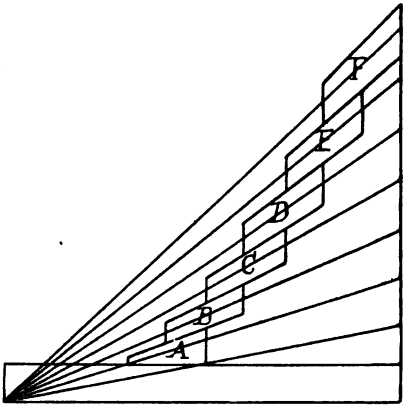


Fig. 29

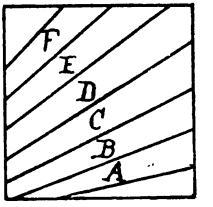


Fig. 30

and D likely to be found together, and these sections are arranged economically as regards space as shown in Fig. 30. On this diagram may be drawn the curve (if the diagram is constructed for half stations)—

$$V_{cy} = \frac{50}{27}(wc + sc^2)$$

from which curve may be taken the center height of a level section having the same volume as any three-level section, if it is desired for any purpose.

To use the diagrams for volumes by the prismoidal formula, the volumes by the average end area method may be taken from the diagram just described, and the difference between the volume by this method, and that by the prismoidal method, may be read from a diagram, specially prepared to show that difference, and applied to the quantity already found by the average end area method.

From Equations (8), (13), and (15) that difference is found to be—

$$C = \frac{l}{27 \cdot 12}(D_1 - D_2)(c_1 - c_2).$$

The diagram would be a series of straight lines, one for each value of either $(D_1 - D_2)$ or $(c_1 - c_2)$, whichever is chosen to

Digitized by Google

represent the tangent of the angle the line is to make with the axis of the other quantity. Or a diagram may be prepared to read the volume by the prismoidal formula directly, and it will be on some accounts more convenient to make such a diagram. From Equations (8) and (15) the volume in cubic yards of a prismoid 100 feet long is—

$$V = \frac{100}{12 \cdot 27}\left[\left(c_1 + \frac{w}{2s}\right) D_1 + \left(c_2 + \frac{w}{2s}\right) D_2 + 4\left(c_m + \frac{w}{2s}\right) D_m\right] - \frac{100\, w^2}{4 \cdot 27\, s},$$

which may be written, remembering that $c_m = \frac{c_1 + c_2}{2}$, and $D_m = \frac{D_1 + D_2}{2}$,

$$V = \frac{100}{12 \cdot 27}\left[D_1\left((2c_1 + c_2) + 3\frac{w}{2s}\right) + D_2\left((2c_2 + c_1) + \frac{3w}{2s}\right)\right] - \frac{100\, w^2}{4 \cdot 27\, s},$$

and this equation is very readily diagramed.

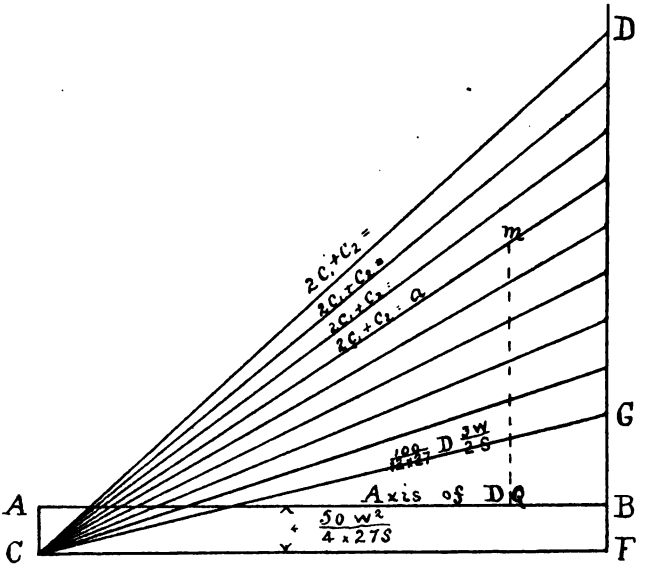


Fig. 31

In Fig. 31, let AB be the axis of D. Since there are two

Digitized by Google

values of D, it will be necessary to enter the diagram twice for the full volume, and for the purpose of construction it will be found more convenient to subtract half the final constant term of the equation from each of the two variable terms, rather than to subtract the whole from the sum of the two variable terms. Lay off from the line AB, downward, a distance $\frac{50\,w^2}{4 \times 27\,s}$ to the scale of V. Draw a line parallel to AB at this distance below it. Assume AB as the largest probable value of D, then ordinates from this line to CF represent $\frac{50\,w^2}{4 \times 27\,s}$. Now conceiving CF as the axis of D, lay off from F upward, to scale of V, the value of $\frac{100}{12 \cdot 27} D \frac{3\,w}{2\,s}$ for the maximum value of D, to G. Ordinates then to the line CG from the line AB will equal, to scale of V, the value of

$$\frac{100}{12 \times 27} D \frac{3\,w}{2\,s} - \frac{50\,w^2}{4 \times 27\,s}.$$

For the greatest assumed probable value of $2c_1 + c_2$ and greatest value of D compute the value of $\frac{100}{12 \cdot 27} D\,(2c_1 + c_2)$, and lay it off on BD from G to D. Draw DC. An ordinate to CD from AB for a given value of D and the assumed greatest value of $2c_1 + c_2$ will give, to scale of V, the value of

$$\frac{100}{12 \cdot 27} D\,(2c_1 + c_2) + \frac{100}{12 \cdot 27} D \frac{3\,w}{2\,s} - \frac{50\,w^2}{4 \cdot 27\,s}.$$

Divide DG for various value of $(2c_1 + c_2)$ as in the previous diagram. For a value of $D_1 = AQ$ and value of $2c_1 + c_2 = a$, the ordinate Qm gives the required volume.

Entering the diagram again for the second value of D, the value of $\frac{100}{12 \cdot 27}\left[D_2\left((2c_2 + c_1) + \frac{3\,w}{2\,s}\right)\right] - \frac{50\,w^2}{4 \cdot 27\,s}$ is obtained. The two added give the total volume in cubic yards for the station. If the volume for a fractional station is desired, multiply the result obtained in the diagram by the fraction of a station for which the volume is required. This diagram may be divided into sections to reduce its size as in the previous diagram.

Digitized by Google

With irregular areas or side-hill areas the whole volume of a station may be assumed divided into elementary volumes consisting of triangular truncated pyramids.

These may be computed by the average end area or prismoidal methods, and either analytically or graphically.

Let a and b be respectively the base and altitude of one section, and a' and b' corresponding quantities at the other end; then by average end areas

$$V = \left(\frac{ab}{4} + \frac{a'b'}{4}\right)\frac{l}{27}.$$

This may be diagramed by separating the terms, giving

$$V = \frac{l}{27}\,\frac{ab}{4} + \frac{l}{27}\,\frac{a'b'}{4}.$$

The axis of abcissas may be the axis of a, and the diagram would consist of as many straight lines as there are to be assumed values of b, as has been previously described. Its use need not be further amplified.

By prismoidal formula we have, remembering that the middle area has for its dimensions, means of corresponding end dimensions,

$$V = a(2b + b') + a'(2b' + b)\frac{l}{12 \times 27}.$$

This may be diagramed precisely as the last case, the difference being that there must be more lines to cover a given maximum value of b, and owing to this fact and the further fact that the tangent of the angle of inclination of the lines is less than in the previous case, a diagram of the same *size* as the average end area diagram can not be read with the same degree of precision.

With these two formulas or diagrams any volume that may be divided into truncated triangular pyramids can be computed.

A diagram for the correction for curvature may be prepared thus—

The equation representing this correction is Eq. (19).

$$V_c = \frac{100}{3R}(d_1^2 - d_2^2)\left(\frac{c}{2} + \frac{w}{4s}\right).$$

For a 1° curve this is in cubic yards

$$V_c = \frac{100\,d_1^2}{6 \cdot 5730 \cdot 27}\left(c + \frac{w}{2s}\right) - \frac{100\,d_2^2}{6 \cdot 5730 \cdot 27}\left(c + \frac{w}{2s}\right).$$

Digitized by Google

For a curve of any other degree, D, the correction is D times that for a 1° curve. A diagram is constructed for one term of the above equation. $V_c = \frac{100}{6 \cdot 5730 \cdot 27} d_1^2 \left(c + \frac{w}{2s}\right)$ is, if d be assumed constant, the equation of a straight line.

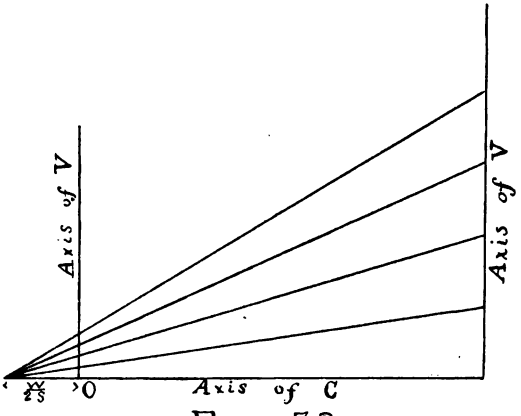


Fig. 32

Assume therefore various values for d and plot the corresponding straight lines. They will all start on the axis of c at the point $V = 0 \quad c = -\frac{w}{2s}$, as may be found by putting $V = 0$ in the above equation. Since the *difference* of the terms in the equation for correction is desired, that portion of the ordinate—for a given value of c—lying *between* the lines corresponding to the given values of d would give the required correction. A diagram for equation (30) is one of the most useful of all diagrams for earthwork. Eq. (30) is—

$$Q = \frac{l}{27}\left(\frac{C^2 2r^2 s}{r^2 - s^2}\right) - \frac{l}{27}\frac{w^2}{4s}.$$

The first term of the right hand member being considered alone gives for a given value of r, a single parabola from which may be taken values of Q for varying values of C (omitting for the time being the constant subtractive term). A number of such parabolas are drawn, one for each assumed value of r. The values of r would be assumed sufficiently close together to give a working diagram, say one for each degree of angle. Slopes under 3° may be neglected and the work considered as

Digitized by Google

level, and the parabola for level cross sections may be drawn on the diagram, when there will be a complete diagram for preliminary estimates. One complete diagram must be made for each value of s used. Since in preliminary estimates the quantity in each station is not required, but simply the quantity in each cut or fill, the constant subtractive quantity neglected in making the diagram, and which is the grade prism, may be summed for the entire cut or fill and subtracted from the final result obtained for the cut or fill on the diagram. Or the horizontal line whose ordinates represent this value may be drawn on the diagram, and measurements taken from it instead of from the axis of C.

The various parabolas may be drawn for slopes as given (*i. e.*, the co-tangents of the angles), expressed in feet, fall or rise per 100 feet, or horizontal distance per 5 feet fall or rise, *i. e.*, the horizontal distance between contours. Thus if contours are drawn on a preliminary map, 5 feet apart vertically, a parabola for each of the following horizontal distances between contours may be drawn, 100 feet, 75 feet, 50 feet, 40 feet, 30 feet, 25 feet, 20 feet, 15 feet, 10 feet. More lines may be used if these are too far apart for close interpolation. A diagram made in this way would be convenient for estimating from a location drawn on a contour map. For an estimate made from notes of cross slopes taken in the field with the clinometer and given in degrees, the diagram should be drawn for degrees. It may be made for even degrees by remembering that in Equation (30) r is the co-tangent of the angle of cross slope.

A diagram for double slopes may be made by the use of Equations (32) and (33), assigning different values for r and considering one of them as negative, and drawing each portion of the diagram for half sections. The two portions of the diagram will lie one on either side of the line corresponding to level cross section.

Diagrams may be constructed for side-hill sections, but it is believed that solidities due to these as well as irregular section solidities, may be taken from a diagram of triangular prisms made in precisely the same way as that shown in Fig. 23 and from the equation

$$V_{cy} = \frac{l}{27} \frac{xy}{2}.$$

Digitized by Google

Thus the work of computing almost anything from more or less complicated formulas may be greatly reduced by the aid of diagrams.

Lest it shall not have been sufficiently apparent from what has preceded, it is again stated that from the nature of the case, in earthwork, no assumption will result in an exact measurement of the amount of material moved, and no measurements likely to be made can give the exact quantities in such irregular work. The best that is ever attempted is to secure a close approximation to the result.

In the discussion of the prismoidal formula it has been assumed that the dimensions of the middle area are means of the corresponding end dimensions. This assumes the surface to be a series of hyperbolic paraboloids, or planes with corresponding points in each cross section. While this is probably never true, it is almost always so nearly true as to warrant the assumption.

If the prismoidal formula is to be used it is desirable to take cross sections in the field, particularly in irregular work, with the view of determining the middle area from them, or the middle area may be itself taken in the field.

It is not intended to imply that, because the work cannot be exactly measured, a rough guess is as good as careful measurements and computations; but it is desired to impress on the student the fact that the field work should be carefully done, and when once done, the computations should be made by that method which, with the least labor and expenditure of time, will give results of precision equal to that obtained in the field-work and the actual construction.

It is not considered, by the writer, necessary or desirable to make computations for earth-work closer than to the nearest cubic yard. While the theory of probabilities indicates that the fractions discarded will not exactly balance, it is nevertheless true that the resulting error will be too small to be worth considering.

The writer desires also to state that while there is a popular prejudice against the doing of anything "just well enough," that is, nevertheless, in almost every case, just the thing to do. The only thing to look out for is to *know*, in any given case, what "just well enough" is.

Digitized by Google

DETERMINATION OF THE PROPER SIZE OF THE

SUPPLY MAIN for the NEW WATER SUPPLY

OF GALVESTON, TEXAS.

By Wynkoop Kiersted.

M. AM. SOC. C. E.
M. RENS. SOC. C. E.

It has occurred to the writer that a brief explanation of an investigation which he, as chief engineer, recently made for the purpose of determining the most advantageous size of the supply main for the new water supply of the City of Galveston, might possess some points of interest to some of the members of the Rensselaer Society of Engineers, therefore, he ventures to present the matter as concisely as possible.

The writer's report on this work, sent you some time since, outlines in a general way the plans of the proposed work. It is, therefore, not necessary to repeat what is therein stated further than may be needed for the purpose of rendering clear the subject matter of this paper.

At the time that this investigation was made, the site of the water supply station, which was to be on the main-land, had not been positively located; consequently, it was assumed to be distant from the distributing pumping station, which is a part of the present water works system, 95,000 feet, a distance that is about 1,700 feet less than that to the supply station site as afterwards located. The assumption was sufficiently near, however, for all the practical purposes of the investigation.

Digitized by Google

It was proposed that the supply main should run in an almost direct line from the supply station to the main-land side of Galveston Bay, a distance of about 11 miles; that it should pass under the bay at approximately its narrowest cross section, about 2 miles in width; and that it should then traverse the island on which the city is located to the storage tank at the distributing pumping station, a distance of about 4¾ miles. No water is to be distributed to the city directly from this supply main. It was proposed that there should be a sufficient storage of water at the distributing pumping station to provide for the fluctuations in the rates of water consumption, thus permitting water to be pumped through the supply main at a uniform daily rate. The storage tank now in use is of iron, 100 feet in diameter, 20 feet high and with its top at an elevation of 21 feet above mean low water in the Gulf of Mexico, which elevation is about the same as that of the natural surface of the ground at the location of the supply station on the main-land.

It is, therefore, evident that the frictional resistance of flow through the supply pipe practically fixes the head, when the storage tank is full, against which the pumping engines at the supply station must work.

No allowance was made for suction lift and frictional resistance of flow through the suction pipes and engines, since the natural head of the artesian waters can be so utilized as to offset these effects.

Previous to this investigation, estimates of cost had contemplated the use of only 36″ pipe, excepting the two miles of pipe under the bay which had been assumed to be 30″ in diameter.

The results of the writer's work are embodied in the tabulations and diagrams herewith presented.

Table I. gives the velocities of discharge, slopes per 1,000 feet of the hydraulic gradient, heads due to frictional resistance of flow through the pipes, all computed for variously assumed rates of discharge through lines of 30″, 36″ and 24″ pipe lines of cast iron or through a combination of these sizes. Column 1 this table gives the rates of discharge in million gallons per day; columns 2, 3 and 4, the velocities of flow; columns 5, 6 and 7, the slopes of the hydraulic gradient per 1,000 feet; column 8, the total resistance head in 95,000 feet of 30″ pipe;

Digitized by Google

column 9, the total resistance head in 33,600 feet of 30″ pipe and 61,400 feet of 36″ pipe; column 10, the total resistance head in 10,600 feet of 30″ pipe and 84,400 feet of 36″ pipe; column 11, the total resistance head in 95,000 feet of 24″ pipe.

TABLE I.

1	2	3	4	5	6	7	8	9	10	11
Discharge in Mill. Gals. Per Day.	Velocities in Feet Per Second.			Slope per 1,000 feet of Hydraulic Gradient.			Total Resistance Head of Entire Pipe Line of 95,000 Feet.			
	30″	36″	24″	30″	36″	24″	30″	30″–36″	30″–36″	24″
5	1.57	1.09	2.46	.43	.16	1.49	41	24	19	142
6	1.90	1.31	2.95	.64	.23	2.13	61	36	26	202
7	2.21	1.53	3.44	.86	.31	2.92	82	48	35	277
8	2.52	1.74	3.93	1.12	.40	3.81	106	62	46	362
9	2.84	1.97	4.42	1.43	.52	4.81	136	80	58	457
10	3.16	2.19	4.91	1.76	.64	5.64	167	98	74	564

In these determinations Kutter's formula ($V = c\sqrt{RS}$) was used, assuming the values of c to be for 36″ pipe, 100; for 30″ pipe, 96, and for 24″ pipe, 91, as graphically determined by the diagram contained in Ganguillet and Kutter's "Flow of Water in Rivers and Other Channels," as translated and enlarged by Messrs. Hering and Trautwine, using a value of (n) equal to .014.

It may appear that this assumed value of (n) is rather excessive, since the values .011 and .012 are frequently employed in determining the velocity of discharge from clean cast iron water pipes. But, after examining all the available data upon the subject, the writer concluded that the assumed value of (n) would probably represent with more correctness than would the other stated values, the condition of water pipes after a few years use, when the interior surface would have become more or less roughened by corrosion and tuberculation; and that it would give at that time a velocity of flow for given slopes and sizes of pipes more nearly the true velocity.

Table II. gives the amount of fuel consumed per day and its cost at the rate of five dollars per ton of 2,000 pounds in delivering the stated quantities of water to the distributing pumping station through 95,000 feet of 30″ pipe by pumping

Digitized by Google

engines of various duties in foot pounds, as determined by the following formula, namely:

$$C = \frac{H \times P \times 8.34 \times 100}{D}, \text{ whence:}$$

H = total resistance head as found in Table I.

P = the amount of water in gallons pumped per 24 hours.

D = the duty of pumping engines in foot pounds per 100 pounds of coal.

C = the daily coal consumption in pounds.

8.34 = weight of U. S. gallons of water.

Tables III., IV. and V. show relations similarly determined of daily fuel cost to pumping engine duties, respectively, for the combinations of 33,600 feet of 30″ pipe and 61,400 feet of 36″ pipe; of 10,600 feet of 30″ pipe and 84,400 feet of 36″ pipe, and of 95,000 feet of 24″ pipe.

From these tables the accompanying diagrams have been prepared, which should be discussed in connection with Tables VI. and VII. and the following subject matter:

In Table VI., column 1 describes the combination of pipes composing the supply main.

Column 2 gives the estimated cost of pipe line and attachments.

Column 3 shows the differences in estimated cost of the several pipe lines with respect to the 30″ line, the signs + and − being used simply to indicate whether the estimated cost is greater or less than that of the 30″ line.

Column 4 represents the interest at 6% (the rate of interest of the bonds which it is proposed to issue for the work of construction) on the difference in costs as given in column 3.

Column 5 gives the daily interest cost.

Table VII. shows the saving in fuel which will result in proportion to the increase of engine duties; results that are deduced from Tables II. and III., and are shown by curves A and B of diagrams. Similar tables of the other combinations of pipes may be prepared, but this typical one partially expresses by figures what the diagrams more thoroughly and comprehensively illustrate graphically.

Digitized by Google

TABLE II.

The daily cost in dollars of fuel for pumping the given amounts of water per 24 hours through 95,000 feet of 30″ cast iron pipe, with engines of various working duties.

Pumping Engine Duty.	5,000,000 Gallons.		6,000,000 Gallons.		7,000,000 Gallons.		8,000,000 Gallons.		9,000,000 Gallons.		10,000,000 Gallons.	
	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.
10	17,097	$42.74	30,524	$76.31	47,872	$119.68	70,723	$176.80	102,082	$255.20	139,278	$348.19
15	11,398	28.49	20,349	50.87	31,914	79.78	47,149	117.87	68,054	170.13	92,852	232.13
20	8,548	21.37	15,262	38.15	23,936	59.84	35,361	88.40	51,041	127.60	69,639	174.09
30	5,699	14.25	10,175	25.44	15,957	39.89	23,574	58.93	34,027	85.07	46,426	116.06
40	4,274	10.68	7,631	19.08	11,968	29.92	17,681	44.20	25,520	63.80	34,819	87.05
50	3,419	8.55	6,105	15.26	9,574	23.93	14,145	35.36	20,416	51.04	27,856	69.64
60	2,849	7.12	5,087	12.72	7,979	19.95	11,787	29.47	17,014	42.53	23,213	58.03
70	2,442	6.10	4,361	10.90	6,839	17.09	10,103	25.26	14,583	36.46	19,897	49.74
80	2,137	5.34	3,816	9.54	5,984	14.96	8,840	22.10	12,760	31.90	17,410	43.52
90	1,900	4.75	3,392	8.48	5,319	13.29	7,858	19.64	11,342	28.35	15,475	38.69
100	1,710	4.27	3,052	7.63	4,787	11.97	7,072	17.68	10,208	25.52	13,928	34.82

Digitized by Google

TABLE III.

The daily cost in dollars of fuel for pumping the given quantities of water per 24 hours through 33,600 feet of 30″ pipe and 61,400 feet of 36″ pipe, with engines of various working duties.

Pumping Engine Duty.	5,000,000 Gallons.		6,000,000 Gallons.		7,000,000 Gallons.		8,000,000 Gallons.		9,000,000 Gallons.		10,000,000 Gallons.	
	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.
10	10,008	$25.02	18,014	$45.03	28,022	$70.05	41,366	$103.42	60,048	$150.12	81,732	$204.33
15	6,672	16.68	12,008	30.02	18,680	46.70	27,577	68.94	40,031	100.08	54,487	136.22
20	5,004	12.51	9,007	22.52	14,011	35.03	20,683	51.71	30,024	75.06	40,866	102.16
30	3,336	8.34	6,004	15.01	9,341	23.35	13,789	34.47	20,016	50.04	27,244	68.11
40	2,502	6.25	4,504	11.26	7,006	17.51	10,342	25.85	15,012	37.53	20,433	51.09
50	2,002	5.00	3,603	9.01	5,604	14.01	8,273	20.68	12,009	30.02	16,346	40.86
60	1,666	4.16	3,002	7.50	4,671	11.68	6,894	17.23	10,008	25.02	13,621	34.05
70	1,430	3.57	2,575	6.44	4,003	10.01	5,910	14.77	8,578	21.44	11,676	29.19
80	1,251	3.13	2,252	5.63	3,503	8.76	5,171	12.93	7,506	18.76	10,216	25.54
90	1,112	2.78	2,002	5.00	3,114	7.78	4,596	11.49	6,672	16.68	9,081	22.70
100	1,001	2.50	1,801	4.50	2,802	7.00	4,137	10.34	6,005	15.01	8,173	20.43

Digitized by Google

TABLE IV.

The daily cost in dollars of fuel of pumping the given quantities of water per 24 hours through 10,600 feet of 30″ pipe and 84,400 feet of 36″ pipe, with engines of various working duties.

Pumping Engine Duty.	5,000,000 Gallons.		6,000,000 Gallons.		7,000,000 Gallons.		8,000,000 Gallons.		9,000,000 Gallons.		10,000,000 Gallons.	
	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.	Pounds Weight of Coal.	Cost in Dollars.
10	7,923	$19.81	13,010	$32.52	20,433	$51.08	30,691	$76.73	43,535	$108.84	61,716	$154.29
15	5,282	13.20	8,673	21.68	13,622	34.05	20,461	51.15	29,023	72.56	41,144	102.86
20	3,961	9.90	6,505	16.26	10,216	25.54	15,345	38.36	21,767	54.42	30,858	77.14
30	2,641	6.60	4,337	10.84	6,811	17.03	10,230	25.58	14,512	36.28	20,429	51.07
40	1,981	4.95	3,252	8.13	5,108	12.77	7,673	19.18	10,884	27.21	15,429	38.57
50	1,585	3.96	2,606	6.50	4,087	10.22	6,138	15.34	8,707	21.77	12,343	30.86
60	1,320	3.30	2,168	5.42	3,405	8.51	5,115	12.79	7,256	18.14	10,286	25.71
70	1,132	2.83	1,859	4.65	2,919	7.30	4,384	10.96	6,219	15.55	8,816	22.04
80	990	2.47	1,626	4.06	2,554	6.38	3,836	9.59	5,442	13.60	7,714	19.28
90	880	2.20	1,445	3.61	2,270	5.68	3,410	8.52	4,842	12.10	6,857	17.14
100	792	1.98	1,301	3.25	2,043	5.11	3,069	7.69	4,353	10.88	6,172	15.43

TABLE V.

The daily cost in dollars of fuel for pumping 6,000,000 gallons of water per 24 hours through 95,000 feet of 24″ cast iron pipe, with engines of various working duties.

Weight and Cost of Coal Per Day.	Duty of Pumping Engines in Million Foot Pounds.										
	10	15	20	30	40	50	60	70	80	90	100
Weight Coal in lbs.	101,080	67,387	50,590	33,693	25,520	20,216	16,846	14,440	12,635	11,231	10,108
Cost Coal in $ . .	$252.45	$168.47	$126.50	$84.23	$63.80	$50.54	$42.11	$36.10	$31.59	$28.08	$25.22

Digitized by Google

TABLE VI.

1	2	3	4	5
Description of Pipe Line.	Estimated Cost.	Difference in Cost.	Interest at 6%.	Daily Interest Cost.
	$	$	$	$
95,000 feet of 30″ Pipe	650443	. . . . .	. . . . .	. . . .
33,600 ft. of 30″ and 61,400 ft. 36″ pipe	719866	+69423	+4165.38	+11.41
10,600 ft. of 30″ and 84,400 ft. 36″ pipe	747723	+97280	+5836.80	+16.00
95,000 feet of 24″ Pipe	532989	−117454	−7047.24	−19.30

TABLE VII.

Cost saving of fuel for various engine duties and stated rates of discharge in million gallons.

Engine Duties.	5 m. Gallons per 24 Hours.	6 m. Gallons per 24 Hours.	7 m. Gallons per 24 Hours.	8 m. Gallons per 24 Hours.	9 m. Gallons per 24 Hours.	10 m. Gallons per 24 Hours.
	Cost Fuel Saving.	Cost Fuel Saving.	Cost Fuel Saving.	Cost Fuel Saving.	Cost Fuel Saving.	Cost Fuel Saving.
	$	$	$	$	$	$
10	17.7	31.3	49.6	73.4	105.1	143.8
15	11.8	20.8	33.0	48.9	70.1	95.9
20	8.9	15.6	24.8	36.7	52.6	71.9
30	5.9	10.4	16.5	24.5	35.0	47.9
40	4.4	7.8	12.4	18.4	26.3	36.0
50	3.5	6.2	9.9	14.7	21.0	28.8
60	3.0	5.2	8.3	12.2	17.5	24.0
70	2.5	4.5	7.0	10.5	15.0	20.6
80	2.2	3.9	6.2	9.2	13.1	18.0
90	2.0	3.5	5.5	8.2	11.7	16.0
100	1.8	3.2	5.0	7.3	10.5	14.4

We note in Table VI. that the daily interest cost on the additional investment necessary to put in the therein described combination supply main, rather than one having the uniform diameter of 30″, is $11.41, a daily cost that is not equalled as shown by Table VII., by the cost saving in fuel due to pumping against the reduced resistance head of the combination main with an engine giving thirty million foot pounds duty, until the discharge amounts to 6 million gallons per day; or, with an engine giving fifty million foot pounds duty, until the discharge is about 7½ million gallons per day; or, with an engine giving

Digitized by Google

seventy million foot pounds duty, until the discharge exceeds 8 million gallons per day, etc.

This is more clearly shown by referring to the diagrams. It will there be noticed that the intercept of an ordinate at any given point between any two of the curves measures the cost saving or loss (depending upon the method of comparison) of fuel for a certain engine duty; consequently, if the point where the ordinate intercept between the curves *A* and *B* is equal to 11.41, be located, as it is at points *a*, *b*, *c*, *d* and *e* on the several diagrams, it will graphically indicate the relations which have been referred to in Tables VI. and VII.

With soft coal worth $5.00 per ton of 2,000 pounds, and with an engine working under the conditions which will obtain at the supply station, the engine performance should, in the opinion of the writer, afford a duty of at least 65 to 70 million foot pounds; a performance which, as has been stated, is economically limited to the pumping of a little more than eight million gallons per 24 hours through a 30″ main, but which, until that point is reached, will maintain the additional fuel expense of pumping through this size of supply main, rather than the combination main, below the interest on the difference of cost of these two pipe lines.

With respect to the water consumption, it will be noticed that the writer states in his report to the Joint Committee on Water Supply that, "In building public works it is not considered economy to anticipate the probable increase of the population of a city beyond a period of fifteen years. This assumption is a fair one, namely, that at the end of this period the population of your city may reach 100,000 to 120,000 in number and that the daily consumption of water will not for legitimate purposes exceed seventy gallons per capita per twenty-four hours."

The conclusion reached by the writer in the light of the foregoing exhibits was that a supply main larger than 30″ in diameter was unnecessary, until the rate of pumping water shall have exceeded 8 million gallons per 24 hours and the population of the city shall have numbered about 100,000 to 120,000; provided, that at the time these limits are reached, pumping engines performing a duty of 65 to 70 million foot pounds shall be in use.

Digitized by Google

It will be noticed in Table I. that the total resistance head, when a given quantity of water is being pumped through 95,000 feet of 24″ pipe, is more than three times greater than that of a 30″ pipe line of the same length when the same quantity of water is being pumped.

By Table VI., it is shown that the daily interest cost on the difference between the investments required to build the pipe line entirely of 30″ pipe and that to build it of 24″ pipe is $19.30, which amount will be exceeded, as shown by curve *D*, Fig. II, of the diagrams, by the additional daily fuel cost that will result when six million gallons of water are being pumped against the greater resistance head of the 24″ pipe line, until there is a developed engine duty of over ninety million foot pounds. Such a high duty for every day work should not under prevailing conditions, be expected. It is, therefore, obvious that, with due regard to the future requirements of the city, a continuous line of 24″ pipe would be too small and uneconomical.

Combinations of 30″ and 24″ pipes, similar to the combinations of 30″ and 36″ pipes, were estimated upon; but in every instance the result was, in the writer's opinion, unfavorable to the use of any size of pipes smaller than 30″ in diameter.

There are other points of interest exhibited by the diagrams, which will be apparent by a study of them but which, being somewhat foreign to the subject of this paper, are not brought out.

Digitized by Google

NATURAL ASPHALTUM AND ITS COMPOUNDS.*

SYNOPSIS.

PAGE.

I. IMPORTANCE OF ASPHALTUM 167
- (*a*) Number of mines.
- (*b*) Branches of engineering, etc., using it.
- (*c*) Men employed.
- (*d*) Capital invested.

II. DEFINITIONS 168
- (*a*) Asphaltum; Crude, Refined and Asphaltic Cement.
- (*b*) Asphaltic Limestone.
- (*c*) Asphaltic Sandstone.
- (*d*) Asphalt Pavements; Trinidad Asphalt class, Asphaltic limestone class, Asphaltic sandstone class of roadway pavements. Asphaltic mastics for walks, floors, roofs, etc. Asphaltic Concretes, etc.

III. POSITION IN MINERALOGY 172
- (*a*) Group of Bitumens to which asphaltum belongs.
- (*b*) Asphaltic compounds and occurrences by classes, deposits and countries.

IV. ANALYSES AND TESTS 175
- (*a*) Chemical.
- (*b*) Physical.
- (*c*) Detection of coal-tar adulterations, etc.

V. MINING, PREPARATION AND USES 182
- (*a*) Trinidad Asphalt as a standard.
- (*b*) Asphaltic limestone.
- (*c*) Asphaltic sandstone.

VI. HISTORY 187
- (*a*) Ancient times.
- (*b*) Modern times, 1626 to 1893.

VII. STATISTICS 189
- (*a*) Mines, Europe and America.
- (*b*) Pavements, Europe and America.

*Copyright, 1894, by J. W. Howard.

Digitized by Google

NATURAL ASPHALTUM

AND ITS COMPOUNDS.*

By J. W. Howard, B. L., C. E.

I. IMPORTANCE OF ASPHALTUM.

The importance of an industry can be judged by its volume, value, usefulness and the kind and number of men employed. Every engineer, architect and constructor needs at least a general knowledge of the properties and uses of asphaltum.

This paper is intended to give simply and briefly an outline of those scientific and general facts about asphaltum, which are essential to technical pursuits.

The importance of the asphalt industries can be judged first, from the fact that there are more than 65 known mines and deposits; secondly, because it enters into constructions and preparations used in all branches of engineering and architecture, as well as in many manufactures.

There are employed in mining, shipping, refining, preparing and applying asphalt and its compounds, scores of graduates of technical institutions, hundreds of business men and thousands of laborers.

The volume of the industry is measured by the fact that the one branch of this industry, asphalt paving, is found in more than 92 American and 55 European cities. This means an investment of at least $55,000,000 of municipal funds and a working capital, if combined, of at least $15,000,000.

The growth of asphalt industries has been very rapid during the past forty years in Europe and twenty in America.

The men engaged in it have been too busy to write its complete

* Copyright, 1894, by J. W. Howard.

Digitized by Google

history or a full treatise on the subject. Fragmentary articles, pamphlets, etc., in either German, French, Italian or English are available; each touching upon some mine or branch of the work. These are generally issued by commercial companies; while acting as advertisements they contain much practical information. Actual experience with asphaltum and its compounds is the best teacher, especially when guided by the recorded experience of others.

II. DEFINITIONS.

A series of precise definitions makes the best introduction, so that it shall be clearly understood just what is meant by the terms used.

(*a*) *Definition of Asphaltum.—Asphaltum*, called generally in English Asphalt, is the Latin for the Greek ἄσφαλτος (asphaltos).

It is a natural mineral pitch and is practically unchangeable at ordinary temperatures. When pure, or nearly pure, it is dark brown or black. When struck with a hammer at ordinary temperatures it is brittle. Its fracture is dull and conchoidal. It is sectile. When rubbed or freshly broken it emits a bituminous odor. Its specific gravity in its natural state is from .96 to 1.68, due to impurities and porosity. It is not soluble in water. It is soluble in carbon disulphide. It begins to melt at about 190° F. and flows at about 200° F. It is partly soluble in alcohol, turpentine, ether, naptha and petroleum. Its hardness at 70° F. is 2½ to 3 Dana scale.

Asphalt is often called by other names, as Trinidad asphalt, Cuban asphalt, Bitumen, Jews' Pitch, etc.

The word asphalt, with slight changes in spelling, is common to all languages.

While asphaltum is a bitumen, all bitumen is not asphaltum, as will be explained in Sec. III. of this paper.

In Europe certain persons, probably for brevity, have called asphaltic limestone, "Asphalte." Asphalt simply saturates the limestone. This compound of limestone and asphalt should not be called asphalt.

In America, the public has been led into the error of supposing that the by-product of the gas works, namely, gas-tar and

Digitized by Google

coal-tar, were asphalts. This is due to the efforts of either unprincipled or ignorant persons, who have desired to sell coal-tar preparations, which, while they have valuable uses, should not be sold or used where asphaltum is needed. They are the products of distillation. The volatile oils contained in them, if exposed to the air or sun, soon disappear and black powder remains. Real asphaltum retains it fixed oils and cementing qualities, hence its lasting properties in its natural and artificial compounds.

Refined Asphaltum is the product resulting from the refining by heat, at a temperature below 400° F., of crude asphaltum. The water and vegetable matter are removed. Commercially refined asphalt used in engineering, architecture, etc., still contains some earthy matter. It is not necessary to fully remove this, because in the mixtures subsequently used it would be replaced when mixed with sand, limestone dust, etc., as occasion demands.

Asphaltic Cement is a combination of refined asphalt and residuum oil of petroleum of a standard specific gravity at a given temperature. The proportions used vary with the objects for which the asphaltic cement is prepared. From 15 to 18 lbs. of residuum oil is used to every 100 lbs. of refined asphalt. They are mixed hot and agitated by air or mechanical means.

(*b*) *Asphaltic Limestone.*—Asphaltic limestone is a natural compound of asphalt and limestone. Maltha, and other bituminous substances, are sometimes present. Sulphur, sand and a few other foreign substances are generally found in it. Its fracture is irregular, and its color is from brown to chocolate black when freshly broken. The color is darker in the specimens containing the larger amounts of asphaltum. The percentage of asphaltum permeating the limestone varies in different deposits, and in different parts of the same mine, from less than 1% to above 20%. The grain of asphaltic limestone is very fine. Under the microscope the smallest particle is coated with asphaltum. If cut by a sharp blow of an axe it appears grayish white along the cut, due to forcing out of the asphaltum and leaving particles of limestone exposed. If a piece contain above 10% of asphalt it can be warmed and broken in the hands. A piece heated over the fire falls apart. If heated in

Digitized by Google

a pan and held for an hour at a high temperature, the asphaltum is driven off and a gray, powdered limestone remains. A bituminous odor is perceptible in all samples when freshly broken if they contain above 4% of asphaltum.

Asphaltic limestone is sometimes called "rock asphalt" and "bituminous limestone." In France, while often referred to erroneously, as "asphalte," its real name is "calcaire bitumineux." The German name is "Asphaltkalkstein," and for brevity, "Asphaltstein."

(*c*) *Asphaltic Sandstone.*—Asphaltic sandstone is composed of sandstone containing from less than 1% to sometimes as high as 70% of asphaltum. Maltha and petroleum are often present to the injury of the asphaltic sandstone for engineering uses. This material is called by a few people in California by the name "asphalt rock," which is misleading. It has not yet proved itself very valuable. Experiments are being constantly made to establish its value and how to use it.

(*d*) *Asphalt Pavements.*—Asphalt pavements are of three classes, laid upon concrete or other suitable foundation, and composed of either Trinidad asphalt, sand, powdered limestone, etc.; or asphaltic limestone, or asphaltic sandstone, if laid upon roadways. But if laid upon walks, floors, roofs, etc., one or more of these materials are combined in various ways. I will define them separately.

Trinidad asphalt pavement, the standard of this class, is laid briefly as follows: The street is graded, rolled with a steam roller, supplied with a cement foundation, generally five or six inches thick. In case the street is covered with a Macadam roadbed or old stone-block pavement, these are often used as foundations without the hydraulic cement concrete. If Macadam or old stone-blocks are used for a foundation, a thin layer, about 1½ inches thick, of asphaltic concrete, called "binder" is first laid upon these old surfaces. The asphalt paving mixture, called the "wearing surface," is laid upon the foundations, prepared as described. The wearing surface is also laid on 1½ inches of binder spread over 5 inches of hydraulic cement concrete.

This wearing surface mixture is spread with hot rakes and compressed with a steam roller to the thickness desired, 1½, 2,

Digitized by Google

and in certain places, 2½ inches thick, as the street and system of pavement demands. The Trinidad asphalt paving mixture is compounded as follows: Asphaltic cement is first prepared by mixing refined asphalt and residuum oil, using from 15 to 18 lbs. of residuum oil, according to the climate, etc. The asphaltic cement serves to cement together the sand and powdered limestone entering into the wearing surface mixture, which is as follows:

Asphaltic cement, ... 12 to 15% by weight.
Sand . . . . . 83 to 70%.
Powdered Limestone . . . . 5 to 15%.

The mixing is done in a special toothed mixing apparatus, the teeth interlacing and revolving on two shafts in opposite directions. The asphaltic cement and sand are first previously heated to above 250° F., which is the temperature at which the mixture should be laid upon the street. The porportions vary according to the kind of sand used and the climate. The proportions of oil in the asphaltic cement, as well as the proportions of the materials in the paving mixture, vary with the climate. More asphaltic cement is used in cold climates than in warm ones. This easy adaptability to climate is one of the reasons of the success of this pavement.

Asphaltic limestone pavement is a pavement laid upon a suitable concrete or other foundation, and composed of powdered asphaltic limestone spread upon the foundation previously prepared, and compressed in position by hot iron rammers, or otherwise. The process is, briefly, thus: The asphaltic limestone is mined in large lumps, crushed in a suitable crusher, powdered while cold in an appropriate pulverizer. The powder is heated to a temperature a little above 275° F., spread upon the street and compressed in position. The natural asphaltic limestone, called sometimes rock-asphalt, must contain about 10% of asphaltum, more or less, according to the climate, for this class of paving.

Asphaltic sandstone pavement is a pavement of natural asphaltic sandstone, prepared, briefly, as follows: The material is mined, crushed by a crusher, if not fine enough when arriving from the mine, and then disintegrated by the injection of steam into the mass, or by passing the material through a revolving steam

Digitized by Google

heated jacket, or cylinder. The material, when fine, is raked upon the prepared foundation and compressed in position generally by a steam roller. Asphaltic sandstone pavements, because still in an experimental stage, have not yet definite percentages of asphaltum decided upon as final and best.

Asphaltic mastic is of three kinds. First, that which is composed of a mixture of powdered asphaltic limestone, Trinidad asphalt and possibly a little residuum or other oil; second, that which is composed of sand, possibly a little limestone dust and about 22% of asphaltic cement; third, that which is composed of crushed asphaltic sandstone, enriched by the addition of either asphaltic cement or a little maltha. All asphalt mastics are mixed or prepared hot, and generally run into molds varying in shape, at the different factories. These are called asphalt mastic blocks and are generally about six inches thick, and either round, square or hexagonal. They are shipped to the place needed and there remelted and used in making walks, floors, roofs and in many other ways.

Asphaltic Concretes are composed of either crushed stone cemented together with asphaltic limestone mastic; or of crushed stone cemented together with Trinidad asphalt mastic, that is, a mixture of crushed stone, sand, limestone dust and asphaltic cement, mixed hot and compressed in layers by hand. These concrete foundations have many uses.

The details of mining, preparing and using of asphalt and its compounds will be given in Section V., further on.

III. POSITION IN MINERALOGY.

(*a*) *Group of bitumens to which asphaltum belongs.*—It is best to pass at once to the position of asphaltum and its compounds among minerals, omitting the geology and origin of these materials, which form a special field of study and investigation not yet in tangible shape.

Among the hydro-carbons there is a group of minerals called the Bitumens, or bituminous substances, which, arranged in order of their density, are as follows:

Digitized by Google

TABLE NO. 1.

BITUMENS.	1.	*Natural Gas*,
	2.	*Natural Naptha*,
	3.	*Petroleum*, or Mineral Oil,
	4.	*Maltha*, or soft, sticky Bitumen, (at 77° F.,)
	5.	*Asphaltum*, or stiff, non-sticky Bitumen.
	6.	*Glance Pitch*, or dry, brittle Bitumen,
	7.	*Elaterite* and others.

This list suffices to establish the position of asphaltum among bituminous substances. These bitumens are seldom found native or pure, but are mixed two or more together; also in combination with earth, clay, sand, etc. Sulphur, magnesia, carbonate of lime are often present.

(*b*) *Asphaltic compounds and occurrence by classes, deposits and countries.*—The grouping of asphaltic minerals and their distribution in the world can best be shown by means of a tabular arrangement. The following Table No. 2, while not fully complete, is the result of fourteen years' correspondence, reading and visits to asphalt mines. This is the first time that I have ventured to publish the same, and I hope it will help systematize the subject of asphaltum and its compounds. The volume of the industry is so great that standard definitions and systematic arrangement of materials are imperative.

Digitized by Google

Table No. 2.

Important Natural Bitumens.

- **Natural Gas** — Ohio, Pennsylvania, California, etc., in the United States, Russia, France, etc.
- **Natural Naptha** — Found in petroleum districts (of little value, superseded by artificial naptha from crude petroleum.)
- **Petroleum** — Pennsylvania, Ohio, Wyoming, California, etc., in United States, Russia, etc. (Consult books on petroleum.)
- **Maltha** — California, Wyoming, Alabama, Utah, Colorado, Kentucky, New Mexico, Ohio, Texas, Indian Territory, etc., Russia, France, Germany, etc.
- **Asphaltum**
  - Asphaltum almost pure.
    - North America — Utah, California, Texas, etc.
    - Central America — Cuba, Mexico, etc.
    - South America — Trinidad, Venezuela, Peru, Columbia, etc.
    - Europe — Caucassia, Syran on the Volga, Germany, France, Italy, Austria, etc.
    - Asia — Hit on the Euphrates, Asia Minor, Palestine, etc.
    - Africa — Oran in Egypt, probably other places.
  - Asphaltic Compounds
    - North America — West Virginia, Kentucky, Texas, Wyoming, Utah, Colorado, California, Indian Territory, Montana, New Mexico.
    - Central America — Mexico, Cuba, etc.
    - South America — Trinidad (largest supply, most used), Venezuela, Peru, Columbia, etc.
    - Europe — Germany, Switzerland, France, Italy, Sicily, Russia, Austria, Spain, etc.
    - Asia — Asia Minor, Palestine, Bagdad and probably in China.
    - Africa — Egypt and probably elsewhere in Africa.

Digitized by Google

In order to facilitate reference, I have arranged the principal deposits of asphalt and its compounds, by materials and countries or states, in the following :

TABLE NO. 3.

ASPHALTUM.

Mixed With Earthy Matter.	Compounded With Limestone.	Compounded With Sandstone.
Bagdad,	Austria,	California,
California,	California,	Colorado,
Colorado,	Dalmatia,	Cuba,
Columbia,	France,	France,
Indian Territory,	Germany,	Germany,
Mexico,	Hungary,	Indian Territory,
Montana,	Indian Territory,	Kentucky,
Palestine,	New Mexico,	New Mexico.
Peru,	Michigan,	Russia,
Texas,	Italy,	Texas,
Trinidad,	Russia,	Utah.
Utah,	Sicily,	
Venezuela.	Spain,	
	Switzerland,	
	Texas,	
	Utah,	
	Washington.	

Other bitumens, as maltha, etc. ; and impurities, as sulphur, etc., are often present, sometimes to the commercial detriment of the material. One or more mines are worked in each of the above countries, except, perhaps, Columbia, Indian Territory, Montana, New Mexico, Michigan, Washington and Dalmatia.

IV. ANALYSES AND TESTS.

(*a*) *Chemical.*—Without entering into the complicated details of the analysis of hydro-carbons and bituminous substances, I will first call attention to the complete chemical analysis of specimens of asphaltum into their elements from representative deposits in the world. This is shown as follows : in

Digitized by Google

TABLE NO. 4.

Place	Trinidad.	Mexico.	Peru.	Cuba.	Columbia.	Palestine.
Carbon	85.89	80.34	88.67	81.50	88.31	80.00
Oxygen	.56	10.09	1.65	8.90	1.68	.40
Hydrogen . . .	11.06	9.57	9.68	9.60	9.64	9.00
Nitrogen . . .	. . .	. . .	. . .	. . .	.37	10.00
Sulphur	2.49	. . .	. . .	. . .	. . .	.60
Ash	. . .	. . .	. . .	. . .	. . .	. . .
Totals	100.%	100.%	100.%	100.%	100.%	100.%

The above Table of the analysis of specimens of asphaltum is that of chemically refined asphaltum. This sort of analysis, while interesting has very little practical value. The commercial value of an asphalt depends principally upon its physical characteristics, which I will give later. Experience has shown that while chemical analyses are of great value in connection with asphalt industry, nevertheless they must be made upon special lines of investigation. We would proceed as follows: (and I take it for granted that my readers are familiar with the general principles and methods of qualitative analysis, and are able to appreciate, if not personally execute, quantitative analysis), having one or more samples of crude asphaltum to be examined, the first thing to do is to turn to the definition of asphaltum, Section II., heading (*a*), and compare the sample or samples with the characteristics set forth in the definition. See if it remains brittle and retains its form at ordinary temperatures of the air. Is it dark brown to black? Is it brittle when struck? Conchoidal? Sectile? Break it and see if it has a bituminous odor? Specific gravity about 1.00 to 1.68? Is it not soluble in water? Do not be deceived by the clay or other matter being soluble in water. It should begin to melt at 190° F., and flow at a little above 200° F. See if the hardness at 70° F. lies between 2½ and 3 Dana scale.

Dessicate a piece *in vacuo*, keeping the temperature below 212° F. Dissolve out the asphaltum from the crude material with carbon disulphide. Try the solubility of pieces in alcohol of standard specific gravity, in turpentine, ether, petroleum, naptha. By taking a little time, with the assistance of the sand bath, the distillation tests can be made at given temperatures

Digitized by Google

and for given periods of time. It is well to get the ash of each crude sample by combustion. The penetration tests can be made upon different samples of refined asphalt at constant temperatures a little above the temperatures used in making penetration tests upon samples of asphaltic cement.

The expert chemist will think of additional tests. What I have given above is in the way of suggestions to facilitate the comparison in a laboratory of different asphalts.

While the above chemical experiments and investigations are of great value in indicating the advisibility of trying an asphalt from a new source of supply in pavements or other branches of engineering, nevertheless the only safe test is to finally try the asphalt in actual practice, keeping carefully tabulated records of what is done, and finally of the results of the asphalt pavements, mastics, concretes, or other construction into which the new asphalt was introduced.

I will omit the experiments upon asphalts with the spectroscope, in which I am taking great interest. Possibly, in the future, I may try to tabulate the results in this interesting field of asphalt investigation, as yet so little developed.

(*b*) *Physical.*—As each student of science knows, the physical and chemical characteristics of a material are so interlaced that it is difficult to treat of them separately. Hence some of the physical characteristics and tests are contained under heading (*a*) above, because chemistry is necessary to, at least, in part, establish the physical properties.

The physical characteristics of the three principal asphaltic substances, asphaltum, asphaltic limestone and asphaltic sandstone are best given separately.

Trinidad asphalt supplies more than 90% of the asphaltum used in the world. It should therefore be described in detail, being, as it is, a standard asphalt and a type of all. Trinidad, an island at the mouth of the Orinoco, north coast of South America, is the property of Great Britain. Upon this island is found the "Trinidad Asphalt Lake," often referred to as the "Pitch Lake." It is a large, flat crater or deposit of asphaltum, mixed with much earthy and a little vegetable matter and water. This deposit is the property of the British Government. It is about a mile from the Southwest corner of the island, and covers almost 116 acres. The surface is hard, very slightly undulating, intersected by shallow cracks, covered at spots with

Digitized by Google

low temporary vegetation and pools of water. The asphalt is softer as we approach the centre, where it is very soft.

Leaving the description of gathering this material in carts driven upon the deposit, hauling it to the shore, loading it in vessels, refining it in America or Europe, until we reach Section V., where details of mining, etc., will be given, we will pass at once to the description of Trinidad asphalt.

Crude Trinidad Asphalt is dark brown and very brittle at ordinary temperatures. It is easily detached from the surface of the "Asphalt Lake" with a pick. Its irregular fracture shows particles of clay and vegetable matter in it. When warmed in the hand it can be slightly bent. When broken it emits a bituminous odor. It is very porous. The chemical and physical tests indicated above and by the definition of asphaltum are easily made.

The average analyses of crude Trinidad asphalt are as follows:

TABLE NO. 5.

First, by weighing, then driving off the water:
Authority, Richardson.

Water	27.85
Asphalt, earthy and vegetable matter	72.15
	100.%

Second, analyzing the dry, crude asphalt:

Asphalt	52.87
Earthy and vegetable matter	47.13
	100.%

Third, combining results gives average analysis of crude Trinidad asphalt.

Asphalt	38.14
Earthy and vegetable matter	34.01
Water	27.85
	100.%

Finally the complete analyses of the materials contained in crude Trinidad asphalt are as follows:

Authority,	Bowen.	Richardson.
Asphalt	39.83	38.14
Earthy matter	33.99	26.38
Vegetable matter	9.31	7.63
Water	16.87	27.85
	100.%	100.%

Digitized by Google

These are mechanical mixtures. The variation in water is very great. A lump of crude asphalt in a room loses its water in a few weeks. Powdered crude asphalt loses its water by evaporation in a few days. Samples of asphalt from the centre of the "Lake" vary slightly from those near the shore. But for commercial uses the asphalt from within the Asphalt Lake is more constant, reliable and better than that which, for a short time, came from places outside of the "Lake," and was known as "overflow or land pitch." This material contained many variable impurities, and lacked cementing qualities. It is no longer imported into the United States.

Asphaltic limestone has many physical characteristics, due to the various compositions of this material in the same and different mines throughout the world.

The definition of asphaltic limestone, already given under Section II., heading (*b*), indicates many of the characteristics of this material. The analyses of representative asphaltic limestones from four mines is shown in the following:

TABLE NO. 6.

COMPOSITION.	Val de Travers, Switzerland.	Seyssel, France.	Ragusa, Sicily.	Limmer, Germany.
Asphaltum	10.15	8.15	8.92	14.30
Calcium Carbonate . . .	88.40	91.30	88.21	67.00
Clay and Iron Oxide. . .	.25	.15	.91	
Sand	. . .	. . .	.60	17.52
Magnesium Carbonate . .	.30	.10	.96	
Insoluble	.45	.10	. . .	. . .
Loss	.45	.20	.40	1.18
Total	100.%	100.%	100.%	100.%

These analyses are made by the usual solutions in re-agents of qualitative and quantitative analyses. There are many other mines and deposits, varying greatly in the proportions of asphaltum, calcium carbonate and sand present. Traces of many other substances are often found. A little water is generally present; sulphur sometimes appears. It is difficult to distinguish the petroleum and maltha if these happen to be present. This is done, generally, by dissolving out the bituminous substances in the bituminous limestone by means of carbon disul-

Digitized by Google

phide, then separating by evaporation at a very low temperature just above the temperature of the air, the carbon disulphide from the bituminous substances. The separation finally, of the bituminous substances present into petroleum, maltha and asphaltum can practically only be indicated by a series of distillations at given temperatures, increasing the temperatures according to the amounts at which the substances desired are known to be distilled. This analysis of one bitumen from another, that is asphaltum from petroleum, is exceedingly difficult and can only be undertaken by special chemists in well-equipped laboratories.

A piece of asphaltic limestone lying exposed and undisturbed for several months, appears upon its surface of a grayish white color. When prospecting for this material this should be borne in mind, for the undisturbed outcroppings of a ledge or deposit of asphaltic limestone is always gray or white. I attribute this to the chemical and physical action of the carbonic acid gas, moisture and air of the atmosphere assisted by the sun. These agencies disintegrate the surface of the limestone, leaving it covered with a light gray film.

Asphaltic sandstone is found in nature sometimes dense and sometimes soft. The grain varies in some mines from very fine to others where it is very course. Small particles of clay and other substances are often present. Little shells, of several kinds, are frequently found. Its color is almost invariably black. A piece heated upon a stove quickly falls to pieces if above 6% of asphaltum is present. Asphaltic sandstone very often contains petroleum and maltha in large quantities. In America, where crude asphaltum is cheap, it does not pay to extract the asphaltum from the asphaltic sandstone, as is done at a few places in Europe.

(*c*) *Detection of coal-tar, distillates and other imitations and adulterations.*—Just after the success of asphalt pavements in Europe and the consequent reports sent to this country, a desire arose to have smooth and noiseless pavements in American cities. At that time the public did not know what asphalt was, and because of the expense of transporting asphaltic limestone from Europe to America, many artificial compounds and imitations were proposed. Before long that which was previously known as gas-tar or coal-tar began to be sold under the name "asphal-

Digitized by Google

tum." Tar was mixed with sand and gravel, first for walks and then for roadways. These mixtures were called "asphalt walks" and "asphalt pavements." This went so far that at the time of the Tweed ring in New York City, many streets were spread over with coal-tar imitations of asphalt pavements, which were soon called "tar poultice pavements" and rapidly disintegrated. The use of coal-tar preparations for pavements of streets has almost ceased. It is still too extensively used for roofs in this country, whereas, in Europe it is not used at all, except, perhaps, in cases of small temporary sheds where a temporary substance, like coal-tar, is admissible.

The real names of these coal-tar products, manufactured by re-distilling the by-products from the gas-works, are "coal-tar distillates," and they are numbered distillate No. 1, 2, 3, etc., according to their comparative density. A name sometimes given them, probably to hide their origin, is "bituminous cement."

These artificial coal or gas-tar products have legitimate uses; but they cannot take the place of natural mineral asphaltum. Coal-tar must not be used as an adulteration of asphalt or its compounds.

Asphalt and coal-tar differ in origin, composition, odor and fracture. A general similarity in appearance deceives a few, especially when some asphalt is mixed with the tar.

"Paving-pitch" is another name given to coal-tar, especially when it is used for pouring between granite paving blocks, and otherwise in pavements. Coal-tar, a product of distillation of the gas-works, when exposed to water, air or the sun soon loses by evaporation its volatile oils and becomes a dry powder.

Genuine natural asphaltum retains its oils, hence its lasting qualities in pavements, concretes, etc.

A practical method to tell asphalt from coal-tar is as follows: Apply to any gas-works or coal-tar roofer and get a piece of coal-tar. Then apply to some dealer in natural asphaltum for a sample of refined Trinadad or other asphaltum. Having two samples, one of coal-tar and one of asphalt, place them side by side. The coal-tar emits a sharp, acrid odor, easily perceived. The asphalt must be rubbed hard to emit a strong odor. The asphalt odor will be quickly learned. It is entirely different from the coal-tar odor. The asphalt odor is known as a

Digitized by Google

"bituminous odor." It is rather agreeable, perhaps a little sour smelling. Crude and refined asphalt, when quite cold, emits a weak, clay-like odor. Next taste one sample, then the other. Then attempt to chew a little of the refined asphalt. It will crumble. If it be an "asphalt paving cement," made from genuine asphalt and residuum oil of petroleum, a fairly agreeable chewing gum is the result. Any attempt to chew coal-tar will result in disaster to the teeth, for it is very sticky and hard to remove from the teeth.

Next turn to the definition of asphaltum Section II., and to the analyses and tests of asphaltum Section IV., and apply the tests there suggested. The differences between gas-tar and asphalt will be at once apparent.

Expert analysis of mixtures to detect any coal-tar present, if present in small quantities, is exceedingly difficult, but when large amounts of tar are present in the mixture, they are quickly discovered by the sharp odor and otherwise. Much pavement was laid in one or two cities under the name "vulcanite," which was a mixture of about 25% of asphalt and 75% of coal-tar, mixed with sand, powdered limestone, etc. This material is no longer laid where engineers have full control as to what shall enter into the pavements, unless they are influenced by other reasons than technical ones

Asphaltic cement is generally composed of refined asphaltum and residuum oil of petroleum, but at some places a mixture of asphaltum and maltha is being tried, with the promise of possible success. The questions of sufficient and constant supply of a standard grade of maltha enter into the problem.

This mixture of asphalt and maltha should not be regarded as an adulteration, simply an experiment.

Mixtures of asphalt and crude petroleum or asphalt and certain shale oils are not suitable for engineering purposes and must be avoided.

V. MINING, PREPARATION AND USES.

It is best to treat of the mining, preparation and uses of the three substances separately, because they are mined, prepared and used in different ways.

(*a*) *Trinidad asphalt as a standard*, comes from the Island of

Digitized by Google

Trinidad, and from the Trinidad Asphalt Lake upon that Island, as described in Section IV. under heading (*b*).

The carts and horses used to haul the crude asphalt from the deposit to the shore, drive upon the "Pitch Lake." The material, which has already been described as dark brown, brittle and similar to dense, dry peat, is picked loose, thrown into the carts, hauled to the ocean beach, loaded upon scows and lighters and thence into the steamers and sailing vessels which bring it to America or take it to Europe.

The excavations made on the surface of the deposit soon disappear by being filled up with new material from the sides and bottom.

Upon arrival at a refinery in the United States or Europe it is either unloaded and stored up for future use, or conveyed directly into large refining tanks, called "asphalt stills," which are, in brief, cylindrical retorts set in brick work and heated by indirect fire.

The crude asphalt is heated somewhat above boiling water and not above 400° F., for several days. By a few improvements this refining has been reduced to half the time it used to take. This process drives off the water, certain light volatile oils and gases. The vegetable matter floats and is disposed of. A part of the earthy matter settles to the bottom. The product is refined Trinidad asphalt and is drawn off into light barrels with one end out. This refined asphalt is sent by rail or boat from the several refineries to the many cities using it.

A full description of refined asphalt has been given in Section II., heading (*a*), under its definition.

The uses of refined asphalt are many and varied. In engineering constructions, generally speaking, it forms the basis of asphaltic cement, although sometimes used otherwise. This asphaltic cement is valuable where a cementing material is needed which is not affected by water, jarring, slight movements and weather temperatures, when used in connection with other materials, in pavements, bituminous foundations for steam hammers, dynamos or other rapidly revolving machinery. For the same reason it is valuable for roofing, especially where the roof is needed for a promenade or other foot traffic, for upper floors of buildings needing durable and water-proof floors, as in stables with the horses upon the

Digitized by Google

upper floors, granaries, etc., in connection with water-proof tunnels under rivers, as at Chicago, in powder magazines and other military constructions where earth-works cover the masonry, in reservoir linings, in natatorium linings. One of the most valuable applications of asphaltum, and more valuable than realized at first thought, is in connection with "damp courses" in buildings. A damp course, I may explain, is a course of asphaltic cement, either alone or in connection with fine hot sand or gravel, laid over the entire upper surface of a foundation on a line just above the earth. The first layer of bricks, preferably heated, is laid upon this damp course, which thus intervenes between the foundation and the upper construction of a building, preventing moisture from below ascending the walls, as takes place in almost all buildings when there is no damp course to prevent it. If, in addition to this, the outer surface of the foundation is coated with asphaltic cement and the cellar floor covered with, first, a little hydraulic cement concrete and then a layer of asphaltic mastic, a thoroughly dry building results.

Asphalt, as well as maltha and gilsonite, are valuable as a basis for coatings for iron pipes. They are not slowly soluble in water, as is the case with coal-tar products. All specifications for the dipping of iron pipes and many other kinds of iron should demand that this be done in hot natural asphaltum, or its equivalent, made from gilsonite, asphalt, maltha or some mixture of these to suit the case.

Electric, mining, mechanical and naval engineering consume large quantities of asphalt.

With a knowledge of the properties of asphaltum and its compounds, the working engineer will be enabled to produce results which otherwise would perplex him beyond measure.

Trinidad asphalt pavements absorb more than 90% of the asphalt imported from Trinidad. This pavement is laid, briefly, as follows: The street is graded, rolled with a steam roller, supplied with a suitable foundation and then covered with a layer of a mixture of sand and powdered limestone, held together by asphaltic cement. A full description of Trinidad asphalt pavement has already been given under Section II., heading (*d*).

The volume and importance of this class of pavement can be

Digitized by Google

judged from the fact that it is found in 72 American cities, covering an area of almost 14,000,000 sq. yds., on January 1st, 1894, equivalent to more than 911 miles of roadway, laid in sixteen years by thirty contractors or companies.

(*b*) *Asphaltic limestone* is mined either by open quarrying or by tunneling. In warm weather open quarrying is difficult because of the tenacity of the material. Tunneling is carried on sometimes by drills and wedges alone, but generally with powder. Gas is not to be feared in most asphaltic limestone mines. Water, however, often appears and must be constantly pumped out. It is mined in large lumps, crushed generally in a rotary, toothed crusher into pieces three inches and less in size. These pieces are passed through a pulverizer, generally the centrifugal type, sometimes in cylinders revolving and containing loose cast steel balls. The resulting powder is about as fine as mustard seed. It is stored up, preferably under cover, for future use, unless needed at once.

One of the chief uses of this powder is for laying compressed *asphaltic limestone pavement*, known generally as "rock asphalt pavement," and like other pavements into which asphaltum enters, it is called, simply, "asphalt pavement." While technically incorrect, practically it is proper to call a pavement containing asphaltum, an asphalt pavement, because though asphalt forms but from 10% to 15% of the mixture, it is the characteristic which determines the name of the pavement.

The process of laying asphaltic limestone pavements is, briefly, as follows: The powdered material is heated to about 275° F., either upon flat hearths or in revolving heaters. It is hauled to the streets in ordinary wagons, raked to such a depth over its concrete foundation that, when compressed by heated rammers or otherwise, it will have the required thickness, generally two or two and one-half inches. This form of asphalt pavement for streets is not popular in the United States, and appears at but few places in cities near the Atlantic Coast.

Asphaltic limestone street pavements, like all limestone, become polished and prove somewhat slippery under heavy fog or a light rain. They are also slippery in clear, cold, dry weather when clean. The class of asphalt pavement known as "Trinidad asphalt pavement," and others made on this system, are not slippery at any time, except when covered with a sheet

Digitized by Google

of ice. This is due to the fact that Trinidad asphalt pavement is composed of about 80% of sand, making it gritty and not susceptible of becoming polished. Paris, London and Berlin keep large supplies of sand at several points near the streets paved with asphaltic limestone, for use during times of fog, slight rain, snow and ice. This is not necessary and is not done upon streets paved with Trinidad asphalt pavements, nor with pavements composed of asphaltic sandstone in California.

Asphaltic mastic, called often simply asphalt mastic, is made as follows: The powdered asphaltic limestone is heated in revolving retorts, refined Trinidad asphalt is added until the mass contains about 20% of bituminous substances, including the original asphaltum, etc., in the asphaltic limestone. The melted mass is constantly stirred until run off into moulds. The mastic blocks or cakes thus formed are shipped all over the world.

The preparation of the mixture for sidewalks, floors, roofs and other applications of asphaltic mastic is as follows: Put a little refined Trinidad asphalt in a large kettle, about 7 lbs., when melted add 100 lbs. of mastic, stir while heating. When soft add sand or fine gravel little by little up to 60 lbs., as the nature of the work demands. Where necessary, several hundred pounds, even thousands of pounds can be mixed at one time. The amounts given indicate the proportions. The mixture, when ready, is taken from the melting kettles, spread upon the surface to be covered by means of thick wooden spatulas or trowels and rubbed down with pressure, by hand. The surface is finally covered with fine white sand, which is also rubbed over the finished walk, floor, etc.

Asphalt mastic, when warm, is like a soft mortar, and can be used in many ways to great advantage. When cold it is slightly pliable and thoroughly water-proof. It can be thickened by the addition of fine sand or powdered limestone for uses in places subject to the direct heat of the sun, etc.

This form of asphalt mastic is used upon the roofs of all modern public buildings of Europe, and in many of America, where the roofs are flat or almost flat. It is the official sidewalk of the streets of Paris, where almost 1200 miles exist. One reason for this is because most of the pipes being under the

Digitized by Google

sidewalk the asphalt walk is easily removed and quickly replaced. It is without joints and easy to walk upon.

Asphaltic mastic made from asphaltic limestone, asphalt, etc., as well as that made from fine sand, powdered limestone and Trinidad asphalt, is used in almost every branch of engineering and architecture.

(*c*) *Asphaltic sandstone*, the third class of natural compounds of asphaltum and other minerals, has as yet but little value. It is quarried generally by blasting. It is seldom, if ever, mined in tunnels. The definition of this material found in Section II., heading (*c*), shows that it is very variable. The additional characteristics of this material already described in Section IV. at the latter part of heading (*b*), indicate the possible uses, not yet fully determined, to which asphaltic sandstone may be applied.

Asphaltic sandstone pavements are laid as indicated in Section II., heading (*d*), under the definition of this class of pavement.

VI. HISTORY OF ASPHALTUM AND COMPOUNDS.

(*a*) *Ancient times.*—The briefest reference to ancient times will suffice. Asphaltum is mentioned by the Arabic writers under the name" chemar," by the Greeks ἄσφαλτος (asphaltos), by the Latin writers as asphaltum, and sometimes by its generic name, bitumen.

It was used in Egypt in connection with embalming the dead, as found in mummies. They got their supply, in part, from Is, modern Hit, near Babylon and near modern Bagdad, which mines are still worked. Refined asphaltum 60 minæ in weight was a part of the annual tribute sent during the years just previous and subsequent to 525 B. C. from Babylon to Egypt.

The Bible, in Genesis, relates in connection with the Tower of Babel (near Babylon), as follows: "Asphaltus fuit eis vice coementi" (asphalt was used by them for cement). Concerning Noah's Ark we read: "Bituminabis eam bitumine," which reads in the English Bible: "Pitch it with pitch."

There are many places where the substance is mentioned in the Greek and Latin Bibles under the names indicated. The

Digitized by Google

use of asphalt was clearly known in ancient times, as indicated by the writings of Pliny, Virgil, Curtius, Ovid and others. Herodotus described the use of asphalt in the construction of the walls of Babylon. Diodorus gives details of the use of asphalt for cement, especially as poured upon reeds, laid crosswise the entire thickness of these walls, thus binding them transversely. When we remember that three chariots could drive abreast upon the walls of Babylon, we realize the immense amount of asphaltum consumed in connection with their construction.

The Dead Sea was called in ancient times "Lacus Asphaltites."

(*b*) *Modern times, 1626 to 1893.*—The use of asphaltum became practically a lost art during the Middle Ages. The first mention of the subject, as far as I have discovered, was in 1626 in a German pamphlet upon "Hartz-Erde," (Pitch-earth). Dr. Amiest referred in 1692 to the deposits in Val de Travers, Switzerland. These deposits were subsequently fully described and uses suggested and tried by Dr. Erynis, from 1711 to the time of his death, 1736. He also discovered the asphaltic limestone of Lobsann, Germany. In 1802 the deposits of Seyssel, France, were discovered. The asphaltic limestone from Seyssel, was very successful for many years.

In 1832 Count de Sassenay united the asphalt interests of France and Switzerland. Paris adopted asphalt sidewalks in 1838. The engineer Merian developed compressed asphaltic limestone pavements in 1849. Soon afterward the eminent French engineer M. Leon Malo began his long and successful career in connection with asphaltic materials. Paris received its first successful asphalt street pavement in 1854; London, 1869; Buda-Pesth, 1871; Dresden, 1872; Hamburg, 1872; Berlin, 1873; Brussels, Geneva, Leipzig, Frankfort and others soon afterward.

The American cities were at first deceived, having expressed a desire for smooth, quiet pavements, and having heard of the asphalt pavements of Europe. The spurious article which encumbered our streets previous to 1873, was nothing more or less than coal-tar, sand and gravel.

Because of the difficulty of transporting the asphaltic limestone from Europe to America, inventors were encouraged to seek some durable substitute. Successful experiments made

Digitized by Google

between 1871 and 1873 in Newark, N. J., and New York City with an artificial mixture of Trinidad asphalt, residuum oil, limestone dust and sand, gave rise to the class of pavements now so successful, and called Trinidad asphalt pavements. In 1876 this pavement appeared in Washington as a pronounced success. In 1878 it became the standard pavement of Washington. It has been rapidly adopted by more than 71 American cities. It is being laid at the rate of about 140 miles annually.

Asphalt pavements in some form are found in 92 American cities. Leaving the statistics to the following Section VII., I will state that it is necessary to carefully select the materials and constantly supervise all asphalt constructions. It is necessary to examine very carefully the size and quality of the sand and limestone powder, as well as the asphalt and residuum oil to be used in pavements.

New and experimental asphalt pavements should not be tried at public expense.

VII. STATISTICS.

(*a*) *Mines, Europe and America.*—Inasmuch as detailed statistics are very uninteresting, I will give only summations and results.

A reference to Tables Nos. 2 and 3 under Section III., will show the location of the asphalt deposits of the world.

Inasmuch as no asphaltum is exported from the United States to Europe, it is best to confine our attention to that which is imported into this country or mined here. From 1867 to 1893 inclusive, there were unloaded at United States refineries 488,086 tons (2240 lbs. each) of Trinidad asphalt. During the past few years it has been arriving at the rate of about 70,000 tons of "Lake Asphalt" per year. This asphalt is from within the celebrated Asphalt Lake. From 1888 to 1892 some inferior "land" or "overflow" asphalt from outside of the Lake was imported. The variable impurities and lack of cementing qualities of this land asphalt injures it for engineering work and it is no longer imported.

The importations of asphaltic limestone, principally in the form of asphaltic mastic blocks or cakes, is very variable, rang-

Digitized by Google

ing from 900 tons to 20,000 tons per year. This comes principally from Sicily, France, Germany and Switzerland.

The output of the mines of asphaltic materials of the United States is very difficult to ascertain, because dissimilar materials have been grouped in the straggling reports heretofore obtainable, and because several mines are closed and others opened each year. As a whole, the asphalt *mining* industry of the United States is not a success. The demand for the class and quality of materials produced in our mines is far less than the available supply. Trinidad asphalt coming from a deposit large enough to warrant ample supplies of any amount and of constant quality, is used almost to the exclusion of all other asphalts. However, after careful experiment in due time our asphalt mines may become more valuable.

The total production per year of asphaltum, asphaltic sandstone, etc., from the U. S. mines has been within the last six years at the rate of about 42,000 tons. A reference to Section III., Tables Nos. 2 and 3, will indicate the location of the American as well as foreign deposits.

(*b*) *Pavements of Europe and America.*—I will dismiss this portion of the subject by briefly giving the latest statistics, January 1st, 1894. The asphalt pavements of all kinds in the United States and Canada on this date are as follows:

TABLE NO. 7.

KIND OF PAVEMENT.	Sq. Yds.	Miles.
Trinidad asphalt*	13,900,000	911.
Asphaltic limestone	151,000	10.
Asphaltic sandstone and other asphaltic materials, experimental and otherwise	619,000	41.
Totals	14,670,000	962.

N. B.—The above is street roadway pavements and does not include sidewalks.

*Subject to slight corrections from final 1893 reports.

Digitized by Google

The total asphalt pavements of European cities at the same date were as follows:

TABLE NO. 8.

CITY.	Sq. Yds.	Miles.
Berlin	1,280,796	83.
Paris	401,617	26.
London	370,000	24.
Other Cities	271,000	18.
Totals	2,323,413	151.

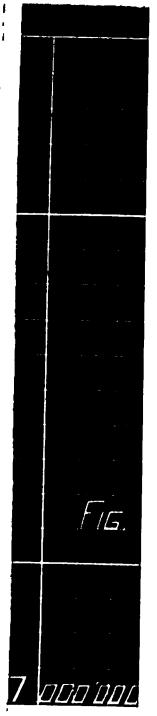
N. B.—In the preceding tables, the roadway is reduced to a common width of 26 feet for comparisons. The roadways themselves vary above and below this width.

The asphalt pavements of America have been laid within twenty years and those of Europe within the past forty years.

The civilization of cities can be gauged by the condition of their streets. The suppression of noise and dirt, made possible by asphalt pavements, in a great measure determines the character and progress of a city.

Digitized by Google

Digitized by Google



Digitized by Google

LIBRARY
OF THE
UNIVERSITY
OF
CALIFORNIA

Digitized by Google

Digitized by Google

Digitized by Google

Digitized by Google

FOURTEEN DAY USE
RETURN TO DESK FROM WHICH BORROWED

This book is due on the last date stamped below, or on the date to which renewed.
Renewed books are subject to immediate recall.

12Dec'55RC

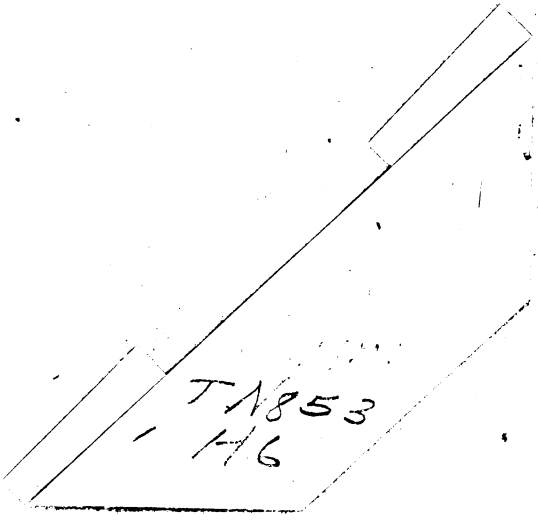
DEC 12 1955 LU

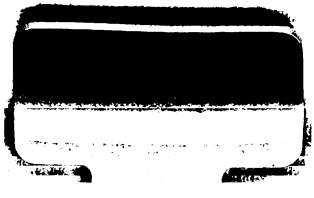
LD 21–100*m*-2,'55
(B139s22)476

General Library
University of California
Berkeley

Digitized by Google

YC 18842





Digitized by Google

Digitized by Google